Bilateral Linkages and the International Transmission of Business Cycles

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Abstract Comovement is more than correlation. This paper estimates the contributions of intermediate production linkages, trade patterns, and financial holdings on the directed graph of linkages that describe the international propagation of macroeconomic shocks at the business cycle frequency. The empirical methodology is a novel generalization of the panel vector autoregression (VAR) which nests both the Global VAR and a VAR with linearly time-varying coefficients. The key novel stylized facts are: 1) trade and financial linkages are associated with transmission of shocks at a lag of one quarter, 2) import and export intensity are asymmetrically associated with shock transmission, and 3) the import of goods used for capital formation has the largest and most significant association with transmission of shocks. In effect, the novel stylized facts in this paper refine the “trade-comovement” puzzle into the “trade-correlation” puzzle and the “trade-transmission” puzzle. I show that both puzzles are challenges for standard international business cycle models, and that an application of sticky information to importing firms can help resolve part of the “trade-transmission” puzzle, especially in combination with other mechanisms that help resolve the “trade-correlation” puzzle.

Keywords: International Business Cycles, International Panel VAR, Trade and Financial Integration, Comovement, International Real Business Cycle Model.

JEL Classification Numbers: F41, F21, E32, C33.

*Address: 115 Morton Hall, 100 Ukrop Way, Williamsburg, VA 23188, USA, telephone: 1-757-221-1389, e-mail: lckent@wm.edu. This draft is preliminary and incomplete; the author welcomes comments. The author is grateful for conversations with Toan Phan, Nick Sanders, Jesper Linde, B.R. Gabriel, and for comments from seminar participants at William and Mary, UNC Chapel Hill, Midwest Macro, and the Richmond Fed. A previous draft of this paper has circulated as “Linkages, Transmission, and the Evolution of International Business Cycles.”
1 Introduction

In the past few decades, the global economy has witnessed both a rapid deepening of international trade and financial linkages and the simultaneous evolution of the interdependence of many countries’ business cycles. What is the relationship between these two trends? In other words, how do the properties of macroeconomic fluctuations change as countries become more deeply linked with their partners?

A natural measure of the interdependence of two countries’ economies is the correlation of their output fluctuations on the business-cycle frequency. But correlation is only one measurement of interdependence. As a pair of countries becomes more linked with each other, the dynamic response of one country to a shock in the other could change as well: to take two extreme examples, news in financial markets transmits almost instantly, while a shock that induces labor to migrate between countries could induce an adjustment that takes years. So, to rephrase the central question more specifically: is there an association between trade of certain goods or holdings of certain assets and the size and speed of the transmission of shocks? This paper establishes new stylized facts to answer this central question using a generalization of methods commonly used in international macroeconomics, then assesses a standard international real business cycle model in light of the new facts.

In contrast to much of the existing literature on business cycle comovement, this paper finds that different sets of linkages explain transmission at different time horizons, with the primary distinction being between transmission within a quarter and transmission between quarters. I estimate the directed graph of linkages that describe the international propagation of macroeconomic shocks at the business cycle frequency. Specifically, I estimate a one-period-ahead forecast of output growth as a linear function of contemporaneous output growth in every country of the world, with the additional assumption that the partial derivative of the forecast of one country’s output with respect to another country’s observed output is itself a linear function of the various low-frequency trends in the trade and financial linkages that exist between the two countries. The coefficients estimated from this regression can answer questions of the following form: How much more sensitive do Canadian output fluctuations become to lagged US fluctuations if Canada imports more investment, intermediate, or consumption goods from the US? How much more sensitive do output fluctuations become if Canada holds more portfolio equity, portfolio debt, or FDI in the US? These questions, posed in terms of one-quarter-ahead forecasts, are equivalent to estimating a nonstructural vector autoregression where the coefficients in the autoregressive matrix vary according to the trends in bilateral linkages between each country. Because of that equivalence, I call this regression a “linkage VAR.” A key innovation in the implementation of the linkage VAR is that it effectively nests as special cases both the Global VAR of Pesaran et al. (2004) and a VAR with time-varying coefficients in the autoregressive matrix.

After having estimated the linkage VAR, I consider the residuals from the regression, which I
interpret nonstructurally as the totality of all international transmission taking place within each quarter. The usual empirical comovement exercise is to regress the set of pairwise rolling-window correlations of output on bilateral linkages; I perform the same exercise on the residuals of the linkage VAR to estimate the determinants of intraquarterly transmission. This regression, together with the linkage VAR, is how I distinguish between the determinants of interquarterly transmission from the determinants of intraquarterly transmission; hence, how I distinguish between transmission taking place at different time horizons.

The regressions show that imports of investment goods and intermediate goods are associated with prolonged and amplified transmission of shocks over time, and to a lesser extent, that FDI holdings are associated with shortened and dampened transmission of shocks over time. Additionally, while cross-border portfolio equity holdings are significantly associated with increased correlation of intraquarterly fluctuations, they are not associated with increased international transmission of shocks at a one-quarter lag. Intuitively, portfolio assets “conduct” shocks faster than trade in intermediates or FDI. This distinction sheds new light on existing discussions about the role of trade vs. finance in comovement, as well as the role of intermediate vs. other goods traded. These results also suggest that any structural model that tries to explain international comovement will need to explain not just unconditional correlation, but also the speed and dynamics of the propagation of shocks.

I show that a standard multicountry real business cycle model cannot replicate the novel stylized facts, even with features known in the literature to increase bilateral correlation of output, such as complementarity of international intermediate inputs and increased correlation of shocks to productivity. Thus, I have shown that the long-standing “trade-comovement puzzle” of Kose and Yi (2006) has two dimensions: first, a “trade-correlation” puzzle relating bilateral trade to increased intraquarterly correlation, and second, a “trade-transmission” puzzle relating bilateral trade to increased interquarterly transmission. I propose a mechanism novel to the international real business cycle setting, sticky information on the part of importers, to help reconcile the “trade-transmission” puzzle separately from the “trade-correlation” puzzle.

I show that assuming sticky information on the part of importers is complementary to mechanisms used to reconcile correlation of output between structural models and data. However, even with the presence of sticky information, there remains a significant gap between model and data. Within the model, the presence of sticky information increases the relationship between trade linkages and interquarterly transmission of value added by a factor of 10, from 0.5% to 5% of the linkage VAR estimate from the data. Furthermore, both the size of the gap between model and data, and the factor by which sticky information shrinks that gap are robust to a range of deviations from the model’s baseline calibration. These results indicate that the well-known puzzle on the weak relationship between trade and comovement within the multicountry real business cycle

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1 Sticky information gives a much larger boost to transmission of shocks to gross output, increasing the relationship by a factor between 30 and 100.
model holds even more strongly when looking at interquarterly transmission.

**Context.** The stylized facts in this paper provide the bridge between two literatures: the empirical and theoretical determinants of comovement between countries, and the international VAR literature.

The first literature estimates the determinants of business cycle comovement, usually through regression of pairwise correlations of output or consumption on observable quantities such as trade volumes and cross-border asset holdings\(^2\). This question has attracted increasing attention as global measures of trade and cross-border asset holdings grow. The usual approach is to explain contemporaneous correlations estimated at rolling windows, using symmetric measurements of bilateral linkages. Since correlation is a symmetric relationship (the correlation of US output with Japanese output is the same as the correlation of Japanese output with US output), the usual explanatory variables, such as in Imbs (2006) and others, are constrained to symmetric linkages such as the sum of trade flows in both directions divided by the product of the two countries’ outputs. Because of this symmetry, this approach cannot distinguish the effects of shocks that start in the US and transmit to Japan from the effects of shocks that start in Japan and transmit to the US. This approach also limits the ability to measure the effects of upstream versus downstream linkages, both for sources and sinks for shocks. This paper goes beyond much of the comovement literature’s estimation of the determinants of correlation to an estimation of the determinants of transmission\(^3\). Estimating transmission directly gives implications for correlations beyond the contemporaneous, and provides a bridge to structural models by showing how impulse responses to shocks change as a function of low-frequency trends.

This literature goes hand in hand with the literature that attempts to reconcile the reduced-form evidence on comovement and transmission with standard structural macroeconomic models\(^3\). Common concerns in this literature has been the differential role of trade linkages versus financial linkages, final goods versus intermediate goods, and the parameters governing elasticities of substitution and access to complete financial markets in determining the strength of transmission. Moreover, almost all of these papers focus on pairwise correlation as the object to be explained. As structural models increasingly ascribe a larger role for financial frictions and a nontrivial banking sector, there is a need to distinguish not only the transmission properties of trade from finance but also the transmission properties of different financial assets from one another. Additionally, as more attention is paid to the determinants of intermediate goods, production networks and how intermediate trade differs from trade in final goods, there is a need to explore the implications

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of intermediate trade for the cross-border propagation of shocks. This project starts down that route by establishing novel stylized facts on the effects of trade in several types of goods (classified by end use) and holdings of several types of assets (portfolio equity, debt and foreign direct investment).

The second literature extends the VAR, a common tool of empirical macroeconomics, to the global environment. There are three main “flavors” to this literature. The first flavor imposes more structure on the global autoregressive matrix by assuming that there are fixed effects on various dimensions. The work by Canova and coauthors typically assumes that the elements of the autoregressive matrix are sums of country, series, and lag fixed effects. In contrast, this paper assumes fixed effects and low-frequency trends, with each specification exploiting a different type of variation in the panel data. The second flavor assumes there are unobserved global and regional factors (which one might identify as global and regional business cycles) and each country’s observed business cycle is some linear combination of these factors and a country-specific component. While this approach goes further than dynamic factor models based on principal components and names the unobserved factors in play, the method relies on an ex-ante partition of countries into regions. While there are numerical methods for finding the partition the data favor most, this paper instead uses measured bilateral linkages to determine the “closeness” of countries and the extent to which groups of countries might share common cycles. The third flavor estimates what in that literature is known as a Global VAR, here taken to have a specific meaning: each country’s vector evolves as a VAR of its own values and a vector of foreign variables. The vector of foreign variables is specific to each country, and is a weighted average of the vector of every other country in the world, weighted by some measure of bilateral linkages. This setup is parsimonious and easy to estimate. This paper extends the Global VAR approach by allowing for a linear combination of many possible weights on foreign vectors, arriving at the linear combination of weights that the data favor most.

Since this approach explicitly models time-variation in the VAR coefficients as a function of observable trends, it also is related to the literature on VARs with time-varying coefficients. The contribution of this paper to that literature is to help identify which of the many factors that vary slowly over time are associated with changes over time in the VAR autoregressive matrix. In that sense, the regression I estimate is a generalization of a VAR with time-varying coefficients.

The structure of the paper is as follows: Section 2 discusses the data and the empirical strategy, with new stylized facts represented both as coefficient estimates and dynamic cross-country responses to shocks. Section 3 lays out a standard international real business cycle and documents that it cannot replicate the novel stylized facts. Section 4 shows that a form of sticky information

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4For example, Canova et al. (2007), Canova and Ciccarelli (2009) and Canova and Ciccarelli (2013).
5See Kose et al. (2003a), Kose et al. (2008), and Kose et al. (2012), for example.

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can improve upon the standard model’s ability to replicate the facts. Section 5 concludes.

2 New stylized facts

This section establishes and elaborates upon a new stylized fact: various trade and financial linkages are associated with business cycle transmission differently at different frequencies.

In what follows, I use both high and low frequency data. The high frequency data are business-cycle frequency components of output, denoted by $y_{it}$ for country $i$ at time $t$. Low frequency data are linear trends of observed series, interpolated to quarterly observations. By construction, innovations to this trend will be orthogonal to innovations in detrended output. I consider trends between each pair of countries within the sample. Between-country trends, such as bilateral imports of intermediate goods or holdings of portfolio assets are denoted $z_{i,j,k,t}$, for countries $i$ and $j$, series $k$, time $t$.

2.1 Data description

Series. High-frequency $y_{it}$ is first-differenced quarterly log real GDP, with the country-specific average removed. Data are taken from the OECD.

I draw the several low-frequency series $z_{i,j,k,t}$ from several sources. None of these are available quarterly, so I first find linear trends in each series, then linearly interpolate the series over quarters as needed. By construction, innovations to the low-frequency trends are orthogonal to innovations of the business-cycle-frequency series.

First, I draw trade series from the OECD STAN Bilateral Trade Database by Industry and End-use (BTDIXE). For each pair of countries in my sample I have four series of exports, where the goods traded are classified by BTDIXE by their final use in the national input-output tables: household consumption, fixed capital formation, intermediate goods and “other.” The BTDIXE database categorizes goods based on the Harmonized System code to fall in one of first three categories, which together cover 90% of all traded goods. The remainder cannot be classified in one of these codes alone and is denoted as “other” exports. Among the goods denoted as “other” exports are personal computers, passenger cars, personal phones, packed medicines, precious stones, antiques and collectibles. I use these categories because it is plausible that trade in consumption, investment, and intermediate goods may respond differently to different shocks, and thus transmission of shocks between a pair of countries might differ depending on which goods are traded more intensely between those countries. At the same time, for the sake of parsimony and clarity of the results, I want to use as few categories as possible. Dividing goods traded into consumption, investment, intermediate, and “other” goods strikes a reasonable compromise between heterogeneity and parsimony.

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8 That is, $y_{it} = \log(Y_{it}) - \log(Y_{it-1}) - \frac{1}{T} \sum_{t=1}^{T} (\log(Y_{it}) - \log(Y_{it-1}))$
Second, I include FDI stocks from the OECD and bilateral portfolio holdings data from the Coordinated Portfolio Investment Survey. As opposed to foreign direct investment, portfolio equity investment entails smaller total shares taken in the target firm (less than 10%) and tends to be more liquid. Since FDI and portfolio equity are substantively different, it is plausible that they respond differently to shocks, and thus entail different transmission of shocks.

Finally, I follow Imbs (2006) and include measurements of industrial dissimilarity between each pair of countries. It is the sum over 2-digit sectors, taken from national input-output tables, of the absolute value of the difference between sectoral shares of GDP in each country.

\[
dissimilar_{ijt} = \sum_{\text{sector}} \left| \frac{y_{i,\text{sector},t}}{y_{i,t}} - \frac{y_{j,\text{sector},t}}{y_{j,t}} \right|
\]  

(1)

In general, there is a trade off between country number and series availability, especially for medium-income countries. Future robustness exercises will explore different points on this tradeoff. Results below are taken for the 58 quarters between 1996:3 to 2010:4, for the 35 countries in the sample. In this sample I am able to include the established OECD countries, countries recently admitted to the OECD such as Korea, Turkey and Mexico, and several large middle income countries such as Brazil, Indonesia, and South Africa. All together this yields 1190 ordered country pairs.

For each bilateral linkage, I construct two measurements: one combined, and one directed. The combined linkages are constructed so that the linkage of type \( k \) at time \( t \) between country \( i \) and country \( j \) is the same as that between country \( j \) and country \( i \): \( \tilde{z}_{i,j,k,t} = \tilde{z}_{j,i,k,t} \forall k,t \). This symmetry is desirable in explaining measurements such as pairwise correlations that are themselves symmetric between country pairs. I construct the combined linkages \( \tilde{z}_{i,j,k,t} \) as follows: for each type of good traded, the linkage is the sum of exports of that good in both directions divided by the sum of the two countries’ GDP: \( \frac{\text{EXPORT}_{i,j,k,t} + \text{EXPORT}_{j,i,k,t}}{Y_{i,t} + Y_{j,t}} \). For each asset, it is the sum of asset holdings of each type in both directions divided by the sum of the two countries’ GDP: \( \frac{\text{ASSET}_{i,j,k,t} + \text{ASSET}_{j,i,k,t}}{Y_{i,t} + Y_{j,t}} \). The measure of industry dissimilarity is already symmetric.

In contrast, I construct the directed linkages such that, generally, \( z_{i,j,k,t} \neq z_{j,i,k,t} \). For each good traded, I construct both \( \frac{\text{EXPORT}_{i,j,k,t}}{Y_{i,t}} \) and \( \frac{\text{IMPORT}_{i,j,k,t}}{Y_{i,t}} \), that is, exports and imports from the perspective of country \( i \), normalized by GDP in \( i \). For each asset, I calculate the \( \frac{\text{ASSET}_{i,j,k,t}}{Y_{i,t}} \), or country \( i \)’s holdings of asset \( k \) in country \( j \), normalized by the holding country’s GDP.

**Summary statistics.** Table I summarizes the low-frequency series, both unilateral and bilateral, combined and directed, at the beginning of the sample and the end of the sample. A few trends are noteworthy. Over the fourteen years of the sample, the per-capita GDP of the countries in the sample have increased relative to the US. On average, the countries in the sample have increased relative to the US.

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9 The full sample of countries is: Australia, Austria, Belgium, Brazil, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, South Africa, Spain, Sweden, Switzerland, Turkey, United Kingdom, and the United States.
Table 1: Descriptive Statistics for low-frequency Variables in 1996 and 2010, in percent

<table>
<thead>
<tr>
<th>Variable</th>
<th>1996</th>
<th>2010</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\mu)</td>
<td>(\sigma)</td>
<td>(\text{Max})</td>
</tr>
<tr>
<td>Combined Bilateral series</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trade / Sum of GDP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>investment goods</td>
<td>0.03</td>
<td>0.06</td>
<td>0.58</td>
</tr>
<tr>
<td>consumption goods</td>
<td>0.05</td>
<td>0.11</td>
<td>1.70</td>
</tr>
<tr>
<td>intermediate goods</td>
<td>0.15</td>
<td>0.32</td>
<td>4.47</td>
</tr>
<tr>
<td>other goods</td>
<td>0.04</td>
<td>0.10</td>
<td>1.03</td>
</tr>
<tr>
<td>Asset holdings / Sum of GDP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>portfolio equity*</td>
<td>0.03</td>
<td>0.13</td>
<td>1.83</td>
</tr>
<tr>
<td>portfolio debt*</td>
<td>0.02</td>
<td>0.23</td>
<td>5.42</td>
</tr>
<tr>
<td>FDI</td>
<td>0.15</td>
<td>0.51</td>
<td>8.05</td>
</tr>
<tr>
<td>Industrial dissimilarity**</td>
<td>0.48</td>
<td>0.18</td>
<td>0.98</td>
</tr>
<tr>
<td>Directed Bilateral series</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exports / GDP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>investment goods</td>
<td>0.06</td>
<td>0.17</td>
<td>3.36</td>
</tr>
<tr>
<td>consumption goods</td>
<td>0.09</td>
<td>0.28</td>
<td>3.98</td>
</tr>
<tr>
<td>intermediate goods</td>
<td>0.27</td>
<td>0.79</td>
<td>13.28</td>
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<tr>
<td>other goods</td>
<td>0.08</td>
<td>0.30</td>
<td>5.47</td>
</tr>
<tr>
<td>Imports / GDP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>investment goods</td>
<td>0.07</td>
<td>0.19</td>
<td>2.67</td>
</tr>
<tr>
<td>consumption goods</td>
<td>0.08</td>
<td>0.23</td>
<td>3.91</td>
</tr>
<tr>
<td>intermediate goods</td>
<td>0.27</td>
<td>0.76</td>
<td>10.27</td>
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<tr>
<td>other goods</td>
<td>0.07</td>
<td>0.21</td>
<td>2.40</td>
</tr>
<tr>
<td>Asset holdings / GDP</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>portfolio equity*</td>
<td>0.05</td>
<td>0.26</td>
<td>4.21</td>
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<td>portfolio debt*</td>
<td>0.05</td>
<td>0.42</td>
<td>12.71</td>
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<tr>
<td>FDI</td>
<td>0.24</td>
<td>1.05</td>
<td>15.54</td>
</tr>
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</table>

* Censored linear extrapolation. CPIS data begin in 1997.
** Levels, not percent
*** Relative to US per-capital real GDP

also become more open. On a bilateral level, trade of all goods and holdings of all assets have increased on average. However, the bilateral amount of trade, when averaged over every pair of countries, is very small, at less than half a percent of GDP. Of the types of goods traded, bilateral trade in intermediates occupied the largest share in 1996 and increased the most over the sample. Bilateral holdings of assets were on the same magnitude of bilateral goods trade in 1996, but has grown far more; in 2010, the average directed bilateral holdings of portfolio equity, portfolio debt and FDI together was almost four percent of GDP.

2.2 Unconditional output growth correlation

The first empirical exercise is common in the literature beginning with [Baxter and Kouparitsas (2005)]: regressing pairwise eight-year rolling-window correlations of detrended output on low-frequency trends in bilateral linkages. The one minor novelty here is the introduction of fixed effects for each country and each country-pair, to illustrate the difference in the association between
linkages and transmission that exists in the cross-section and the association that exists over time. In other words, I estimate:

$$\rho_{Y_{i,j,t}} = \beta_0 + \beta_{i,j,0} + \sum_k \beta_k \tilde{z}_{i,j,k,t} + \epsilon_{i,j,t}$$ (2)

Here \(\beta_0\) is a constant, \(\beta_{i,j,0}\) is a fixed effect for the pair of countries \(i\) and \(j\), and \(\tilde{z}_{i,j,k,t}\) is the average (taken over an eight-year window indexed by \(t\)) of the combined bilateral linkage indexed by \(k\) (for example, trade in investment goods) between country \(i\) and \(j\). Following the literature, the linkages I consider in this regression are combined, not directed. This is a natural assumption when the object of interest, the correlation between output in countries \(i\) and \(j\), is symmetric between countries.

<table>
<thead>
<tr>
<th></th>
<th>((\rho_Y: 1))</th>
<th>((\rho_Y: 2))</th>
<th>((\rho_Y: 3))</th>
<th>((\rho_Y: 4))</th>
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<td><strong>Total trade as % of Sum of GDP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all goods</td>
<td>0.99***</td>
<td>2.44***</td>
<td>1.02***</td>
<td>0.00***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.38)</td>
<td>(0.07)</td>
<td>(0.00)</td>
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<tr>
<td>investment goods</td>
<td>10.94***</td>
<td>1.68***</td>
<td>-0.14***</td>
<td>-0.42***</td>
</tr>
<tr>
<td></td>
<td>(0.46)</td>
<td>(0.23)</td>
<td>(0.05)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>consumption goods</td>
<td>2.28***</td>
<td>-0.42***</td>
<td>-0.10***</td>
<td>-0.42***</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.06)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>intermediate goods</td>
<td>-0.29***</td>
<td>-0.14***</td>
<td>0.05***</td>
<td>0.19***</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.10)</td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>other goods</td>
<td>0.51***</td>
<td>0.32***</td>
<td>0.19***</td>
<td>0.00***</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.10)</td>
<td>(0.03)</td>
<td>(0.03)</td>
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<td><strong>Total assets as % of Sum of GDP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>portfolio equity</td>
<td>0.30***</td>
<td>0.04***</td>
<td>-0.02***</td>
<td>-0.02***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
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<td>-0.04***</td>
<td>-0.06***</td>
<td>0.04***</td>
<td>0.04***</td>
</tr>
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<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>FDI</td>
<td>0.03***</td>
<td>0.02***</td>
<td>0.02***</td>
<td>0.02***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Industry dissimilarly</td>
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<td>-0.00***</td>
<td>0.00***</td>
<td>0.00***</td>
</tr>
<tr>
<td></td>
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</tr>
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<td>No</td>
</tr>
<tr>
<td><strong>R2</strong></td>
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<td>0.52</td>
<td>0.70</td>
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Table 2 gives the result of regression (2). There are a few features to note: First, the inclusion of country-pair fixed effects matters. With the fixed effects, the regression exploits variation within each country pair over time. Without the fixed effects, the regression pools variation along the cross section with that over time. The fact that the two sets of coefficients are so different tells us that the two types of variation are very different. This is intuitive: the cross section includes a set
of countries that are very different in terms of income, industrial structure, and trade composition. It’s not surprising that the variation over time is of a fundamentally different nature than what exists as the initial condition of the sample.

The second feature to note is that the association between output correlation and linkage depends on the type of goods traded and the type of assets held. The second column, with country-pair fixed effects, illustrates this. For an increase in combined bilateral trade in capital of 0.01 percent of a pair’s combined GDP (which is the average increase across all country-pairs in the data between 1996 and 2010), the regression predicts an increase in the correlation coefficient of 0.109. The same increase in consumption goods leads to a much smaller increase in the correlation coefficient of 0.023, while the same increase in intermediate or other goods yields a decrease in the correlation coefficient of -0.003. From this one can conclude that trade of all goods types are associated with changes in pairwise correlation, but with different magnitudes and directions. As for assets, an increase in combined portfolio equity holdings of 0.41 percent of combined GDP (again, the average increase across all country-pairs between 1996 and 2010) yields an increase in the correlation coefficient of 0.123. Interestingly, an increase of the same amount in portfolio debt yields a decrease in the correlation coefficient of 0.016. FDI also yields a decrease of the same magnitude: 0.012. From this one can conclude that there is heterogeneity across the relationships of trade of various goods and asset holdings of various types with output correlation.

2.3 Distinguishing combined from directed, and contemporaneous from lagged effects

Regression (2) conceals many economically interesting mechanisms; there are two basic features of the data one cannot learn from regression (2). First, it does not show what transmission is associated with directed linkages. In other words, one cannot distinguish between correlation of countries’ output and transmission of shocks from one country to another. Second, it does not show which linkages are more or less relevant at different time horizons.

To address the first issue of directed linkages, I construct a second set of bilateral linkages \( z_{i,j,k,t} \) which are directed and asymmetric: generally, \( z_{i,j,k,t} \neq z_{j,i,k,t} \). What additional information does the asymmetry provide? Understanding the transmission of shocks from a Foreign country \( j \) to Home country \( i \), requires understanding the role of various kinds of shocks that might have have asymmetric consequences for exports from \( i \) to \( j \) and imports from \( j \) to \( i \). Therefore, in the set of linkages \( z_{i,j,k,t}^b \) for the purpose of the regression, I include both \( \frac{\text{EXPORT}_{i,j,k,t}}{Y_{i,t}} \), which could be interpreted as capturing the importance of Foreign as an export market for Home, as well as \( \frac{\text{IMPORT}_{j,k,i,t}}{Y_{i,t}} \), which could be interpreted as capturing the importance of Foreign as a supplier of goods to Home. For assets, I consider the measure \( \frac{\text{ASSET}_{i,j,k,t}}{Y_{i,t}} \), or Home’s holdings of asset \( k \) in Foreign, to capture possible balance sheet or contagion effects Home agents may experience when Foreign assets endure shocks.
To address the second issue of linkages at different time horizons, I distinguish between fluctuations that are orthogonal to a forecast made from lagged observables and those that are not. I do this by estimating the conditional expectation function of $y_{it}$ as a function of lagged $y_{jt}$ and the set of bilateral linkages between $i$ and $j$. Specifically, I am interested in a one-period-ahead expectation of $y_{it}$ that is a linear function of lagged output fluctuations at home and abroad:

$$E_{t-1} \left[ y_{it} \mid \{ y_{jt-1} \}_{j=1,..,J}, \{ z_{i,j,k,t} \}_{j=1,..,J,k=1,..,K} \right] = \sum_{j=1}^{J} \alpha_{i,j,t} y_{jt-1}$$ (3)

The coefficients $\alpha_{i,j,t}$ can be estimated by OLS, for example with the additional assumption that the coefficients $\alpha_{i,j,t}$ are not time-variant and the forecast errors $y_{it} - E[y_{it}]$ are orthogonal to the lagged observables. The estimated coefficients in this case describes the conditional transmission between countries. For example, if country $i$ is Canada, country $j$ is the US, then the coefficient $\alpha_{i,j}$ is the partial derivative of the one-period ahead conditional expectation of $y_{it}$ (Canadian output) with respect to $y_{jt-1}$ (lagged US output). The larger $\alpha_{i,j}$, ceteris paribus, the greater the association between lagged US fluctuations and current Canadian fluctuations. In other words, $\alpha_{i,j}$ can be interpreted as the strength of the transmission from US output to Canadian output at a lag of one period.

This paper estimates how these coefficients $\alpha_{i,j,t}$, which describe the strength of transmission, vary systematically across pairs of countries and over time by imposing additional structure on them:

$$\alpha_{i,j,t} = \begin{cases} 
\alpha_{i,i,0} & \text{if } i = j \\
\alpha_{i,j,0} + \sum_{k=1}^{K} a_{k} z_{i,j,k,t} & \text{if } i \neq j
\end{cases}$$ (4)

The additional structure breaks down that transmission among variables that describe low-frequency series $z$. For example, if low-frequency linkage $z_{i,j,k,t}$ is Canadian exports to US / Canadian GDP in 2005, then $a_{m,n,k}$ is the contribution of Canadian exports to US / Canadian GDP in 2005 to the strength of transmission from the US to Canada in 2005, $\alpha_{i,j,t}$. Additionally, there are fixed effects for each country $\alpha_{i,i,0}$ and each country pair $\alpha_{i,j,0}$. Importantly, the dependency of transmission on low-frequency characteristics, that is, the set of $(a_k)$, is constant over countries.

The coefficients $\alpha_{i,j,t}$ have an additional interpretation: as the edges of a directed graph which describes the one-quarter-ahead spillover effects in output growth between countries\footnote{A previous version of this paper considered a specification with multiple series in each country and multiple lags. Due to the length of the available series, one encounters the curse of dimensionality when extending this specification to more series than just output in each country. Bayesian methods exist to overcome this problem, but since this setting is novel relative to standard Bayesian VARs, there is little guidance for the selection of the prior. Future work will address prior selection issues in extending this paper’s analysis to the multivariate setting.} \footnote{Bonaldi et al. (2014) use the same concept in an analysis of linkages between financial institutions.}. Under this interpretation, the coefficients $a_k$ are the weights of linkages on the size of the graph’s edges. To clarify what the edges of this graph represent, consider the following: suppose there is a...
nonstructural shock (i.e. a forecast error) at time $t$. That shock may give rise to innovations in output growth that are correlated across countries. Now consider how that shock propagates, that is, the effects of that shock in time $t + 1$. The conditional expectation of the response of country $i$ to this shock, $E_t [y_{i,t+1} | \{y_{j,t}\}_{j=1..J}, \{z_{i,j,k,t}\}_{j=1..J,k=1..K}]$, will have a component that depends on the response of country $i$’s output in time $t$ (weighted by $\alpha_{i,i,t}$) and a component from each country $j \neq i$ (weighted by $\alpha_{i,j,t}$). The component weighted by $\alpha_{i,i,t}$ describes the internal persistence of the shock within country $i$. The component weighted by $\alpha_{i,j,t}$ is transmission from country $j$ that is not only orthogonal to other foreign countries, but is also orthogonal to the domestic component weighted by $\alpha_{i,i,t}$. So, the edges of the graph which describe linkages between countries are capturing the transmission of shocks between countries that is orthogonal to the internal persistence of those shocks. In other words, we are interested in the graph of the set of spillovers that take place with a delay. This interpretation will be important in understanding how standard business cycle models can or cannot replicate the stylized facts of this section.

Substituting in the restrictions of equation (4),

$$E_{t-1} [y_{i,t} | \ldots] = \alpha_{i,i,0} y_{i,t-1} + \sum_{j \neq i} \left( \alpha_{i,j,0} + \sum_{k=1}^{K} a_k z_{i,j,k,t} \right) y_{j,t-1}$$

(5)

Rearranging,

$$E_{t-1} [y_{i,t} | \ldots] = \sum_{j=1}^{J} \alpha_{i,j,0} y_{j,t-1} + \sum_{k=1}^{K} a_k \left( \sum_{j \neq i} z_{i,j,k,t} y_{j,t-1} \right)$$

(6)

These two equivalent expressions offer some help in interpreting the coefficients $a_k$. On the one hand, writing the conditional expectation in the form (5) emphasizes that each $a$ is the weight on each low-frequency series $z$ and the fixed effects in constructing the coefficients $\alpha$ which describe total transmission. On the other hand, writing the conditional expectation in the form (6) emphasizes the channels through which transmission takes place. The first term $\sum_{j=1}^{J} \alpha_{i,j,0} y_{j,t-1}$ is the time-invariant component. The term $\left( \sum_{j \neq i} z_{i,j,k,t} y_{j,t-1} \right)$ is a weighted sum of business-cycle-frequency fluctuations in output of all of country $i$’s potential trade and financial partners, with the weights being the low-frequency component of the $k$th series of linkages between country $i$ and its partners. Under this interpretation, the coefficient $a_k$ describes the contribution of the $k$th linkage to business-cycle-frequency fluctuations in country $i$’s output. By assuming that the coefficients $a$ do not vary across countries, this specification pools all triple (country pair, time) observations to estimate the between-country channels $a$. I estimate the $a$ using ordinary least squares.

Estimating $a$ in either (5) or (6) via OLS is equivalent to another very common technique in empirical macroeconomics: the vector autoregression. In fact, the regression can be interpreted as the estimation of an unrestricted or nonstructural VAR at one lag with the potential for vari-
ation in the autoregressive matrix over time. Consider what would be the \((i, j)\)th element of the autoregressive matrix:

\[
\frac{\partial}{\partial y_{i,t-1}} E_{t-1} [y_{i,t} | ...] = \alpha_{i,j,0} + \sum_{k=1}^{K} a_k z_{i,j,k,t}
\]

(7)

Similarly, the \((i, i)\)th element would be:

\[
\frac{\partial}{\partial y_{i,t-1}} E_{t-1} [y_{i,t} | ...] = \alpha_{i,i,0}
\]

(8)

If all the \(a\)s are fixed to zero, then the only coefficients are the fixed effects \(\alpha_{i,i,0}\) and \(\alpha_{i,j,0}\). These fixed effects make up the estimated autoregressive matrix in a canonical nonstructural VAR. By introducing the \(a\)s, I allow the elements of the autoregressive matrix to vary over time. Specifically, they vary as linear functions of the set of linkages between country \(i\) and \(j\), in a way that is common across all country pairs. For this reason I refer to the regression in (5) and (6) as a “linkage” VAR, in contrast to a canonical VAR, where the autoregressive matrix is fixed over time. It bears repeating that the VAR I estimate here accords no structural interpretation to any of the residuals; these are forecast errors, not identified shocks.

The linkage VAR is similar to a Global VAR, where foreign fluctuations are weighted by some measure of bilateral linkages. Consider equation (5), and impose the restrictions taken in Pesaran et al. (2004) by fixing the weights of foreign countries on domestic output growth to constants for each country pair \(i\) and \(j\). That is, set \(\alpha_{i,j,0} = 0\), and let there be \(K = J\) bilateral linkages \(z_{i,j,k,t}\) defined as follows, in terms of time-invariant bilateral linkages \(\tilde{z}_{i,j}\):

\[
\text{for all } t, \ z_{i,j,k,t} = \tilde{z}_{i,j} \text{ if } k = i \text{ and 0 otherwise. Then, equation (5) becomes:}
\]

\[
E_{t-1} [y_{i,t} | ...] = \alpha_{i,i,0} y_{i,t-1} + \sum_{j \neq i} \left( \alpha_{i,j,0} + \sum_{k=1}^{K} a_k \tilde{z}_{i,j} \right) y_{j,t-1}
\]

(9)

\[
= \alpha_{i,i,0} y_{i,t-1} + a_i \sum_{j \neq i} \tilde{z}_{i,j} y_{j,t-1}
\]

(10)

The quantity \(\sum_{j \neq i} \tilde{z}_{i,j} y_{j,t-1}\) is usually denoted \(y^*_{i,t-1}\) in this literature, and represents all foreign \((j \neq i)\) contributions to domestic \((i)\) fluctuations. Here, the importance of foreign contributions to domestic fluctuations, \(a_i\), can vary across countries. In contrast, this paper allows multiple types of linkages, and estimates from the data the appropriate weight of each type of linkage in a way that is consistent across countries.

The linkage VAR is similar in another sense to a VAR with time-varying coefficients on the off-diagonal elements. To see this, revisit equation (5), and suppose that there is one low-frequency

\[[\text{In the Global VAR literature, } \tilde{z}_{i,j} \text{ is usually some measure of bilateral trade.}]\]
series \( z_{i,j,k,t} \) for each ordered country pair (that is, \( K = J(J - 1) \) and for each \((i, j)\) there exists a \(k(i, j)\)), and it is a time trend: \( z_{i,j,k,t} = t \) if \( k = k(i, j) \) and 0 otherwise. Then equation (5) becomes:

\[
E_{t-1}[y_{i,t}|\ldots] = \alpha_{i,i,0} y_{i,t-1} + \sum_{j \neq i} \alpha_{i,j,0} + \sum_{k=1}^{K} a_k z_{i,j,k,t} y_{j,t-1} \\
= \alpha_{i,i,0} y_{i,t-1} + \sum_{j \neq i} (\alpha_{i,j,0} + a_{i,j} t) y_{j,t-1}
\]

(11)

(12)

In contrast, what I estimate in this paper is more general: I allow the coefficients of the VAR to vary not just as functions of time but as functions of states, namely, the value of the trend in each type of linkage. Therefore, the regression I pose nests a VAR with linearly time varying off-diagonal autoregressive coefficients as a special case.13

Estimating the \( a \) of the linkage VAR shows what low-frequency series are associated with directed transmission at a lag of one quarter. To get at the transmission that takes place within the quarter, one can turn to the residuals of the linkage VAR regression, or the variation in HP-filtered output that is orthogonal to the linear combination of lagged observables in (5) and (6):

\[
\hat{\epsilon}_{i,t} = y_{i,t} - \hat{E}_{t-1}[y_{i,t}|\ldots]
\]

(13)

These \( \hat{\epsilon}_{i,t} \) represent the sum of all shocks whose impacts are felt in the same period. One can think of the contemporaneous correlation of these \( \hat{\epsilon}_{i,t} \) as representing all transmission mechanisms that take place within the quarter. Repeating the standard regressions of pairwise correlations on combined linkages for correlations of these \( \hat{\epsilon}_{i,t} \) instead of output will shed the same light on what linkages contribute to intraquarterly transmission, as opposed to overall output correlation.

To make the preceding point more concrete, suppose that \( y_t \) is a vector which stacks all the \( y_{it} \). Suppose that \( y_t \) follows an autoregressive process, subject to a vector of shocks \( \epsilon_t \):

\[
y_t = Ay_{t-1} + B\epsilon_t
\]

(14)

The unconditional covariance of \( y_t \), \( \Sigma_y = E[y_t y_t'] \), is a function of the autoregressive matrix \( A \) and the unconditional variance of \( \epsilon_t \), \( \Sigma_{\epsilon} = E[\epsilon_t \epsilon_t'] \). Overall covariance (and hence correlation) arises from both inter- and intra-quarterly transmission, and the covariance of the shocks themselves. This is part of the advantage to estimating a linkage VAR: it can distinguish inter- and intra-quarterly transmission, which is conflated in the regressions which use simple correlations.

---

13 It is also possible to estimate equation (12) under the assumption that all edges of the directed graph of international transmission have evolved with the same time trend: \( a_{i,j} = a \forall i, j \).

14 \( \text{vec}(\Sigma_y) = (I - (A \otimes A))^{-1}\text{vec}(\Sigma_{\epsilon}) \)
### 2.4 Estimation results

<table>
<thead>
<tr>
<th>Home exports to Foreign / Home GDP (%)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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</thead>
<tbody>
<tr>
<td>all goods</td>
<td>-0.04</td>
<td></td>
<td></td>
<td>0.19***</td>
<td>-0.24</td>
<td></td>
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<tr>
<td>investment goods</td>
<td>-0.07</td>
<td>(0.24)</td>
<td>0.19***</td>
<td>-0.24</td>
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<tr>
<td>consumption goods</td>
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<td>(0.17)</td>
<td>0.05</td>
<td>-0.11</td>
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<td>intermediate goods</td>
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<td>(0.06)</td>
<td>-0.00</td>
<td>0.02</td>
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<tr>
<td>other goods</td>
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<td>(0.11)</td>
<td>-0.01</td>
<td>-0.15</td>
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<table>
<thead>
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<th>Home imports from Foreign / Home GDP (%)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tbody>
<tr>
<td>all goods</td>
<td>0.10***</td>
<td></td>
<td></td>
<td>0.89**</td>
<td></td>
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<tr>
<td>investment goods</td>
<td>0.79***</td>
<td>(0.34)</td>
<td>0.09</td>
<td>(0.44)</td>
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<tr>
<td>consumption goods</td>
<td>0.19</td>
<td>(0.25)</td>
<td>0.04</td>
<td>0.27</td>
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<tr>
<td>intermediate goods</td>
<td>0.15*</td>
<td>(0.09)</td>
<td>0.02</td>
<td>0.15</td>
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<tr>
<td>other goods</td>
<td>-0.03</td>
<td>(0.12)</td>
<td>-0.00</td>
<td>-0.05</td>
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<table>
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<tr>
<th>Home assets held in Foreign / Home GDP (%)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
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<tbody>
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<td>portfolio equity</td>
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<td>0.01</td>
<td>(0.01)</td>
<td></td>
<td></td>
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<tr>
<td>portfolio debt</td>
<td>0.00</td>
<td>(0.00)</td>
<td>0.02*</td>
<td>0.01**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FDI</td>
<td>-0.02*</td>
<td>(0.01)</td>
<td>-0.01</td>
<td>-0.01**</td>
<td>-0.02*</td>
<td></td>
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<tr>
<td>Industry Dissimilarity</td>
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<td>(0.00)</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
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<tr>
<td>Low-frequency time trend</td>
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<td>0.00</td>
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<th>Low-frequency country-pair fixed effects</th>
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<th>Yes</th>
<th>Yes</th>
<th>No</th>
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<tr>
<td>$R^2$</td>
<td>0.79</td>
<td>0.79</td>
<td>0.86</td>
<td>0.20</td>
<td>0.79</td>
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</table>


Table 3 gives point estimates for $\alpha$ in the linkage VAR. An important caveat: these are not causal effects. This regression does not make the claim that a given change in a linkage between two countries is exogenous, or occurs in isolation from changes in other linkages. Instead, the regression shows the associations between various low-frequency and high-frequency properties of the data, under the parametric assumptions of the linkage VAR.

How should these coefficients be interpreted? From the point of view of the structural models built by the literature to explain international comovement, both the low- and high-frequency
properties of series simulated from the model depend on a set of deep structural parameters. Variation in these structural parameters would give rise to both variation in low-frequency linkages and variation in propagation. One natural interpretation of the coefficients of the regression is as a benchmark to assess structural mechanisms that can deliver both changes in the low- and high-frequency properties of the data. For example, consider the coefficient on the ratio of Home bilateral exports of investment goods to Foreign / Home GDP under the specification without low-frequency fixed effects. The value is 0.19. This means that an increase of Home’s exports of investment goods by 1% of Home’s GDP, over the range of mechanisms that such an increase happened in the data, is associated with an increase of the response of Home’s GDP growth to the lagged value of Foreign’s GDP growth by 0.19.

Notice that the size of the significant coefficients are larger for trade linkages than for financial linkages in both specifications. This result stands in contrast with other work on the determinants of bilateral correlation that, like Imbs (2006), find that financial linkages have larger effects than trade linkages. To reconcile these results it is necessary to distinguish correlation from transmission. Correlation of output is a function of both transmission across time and the contemporaneous correlation of the shocks that drive output in both countries. Trade could have larger effects for interquarterly transmission, and yet finance could have larger effects for correlation, to the extent that 1) finance drives intraquarterly transmission, i.e. contemporary correlation of shocks, and 2) correlation in output is largely explained by contemporaneous correlation of shocks. The first of these two conditions will be explored in the next regression. The discussion below of a decomposition of the change in output correlation between 1996 and 2010 will offer evidence that the second of these two conditions holds in the data as well.

The inclusion of low-frequency fixed effects allows us to contrast the changes in propagation associated with variation in the cross-section versus the changes in propagation associated with variation over time within each country pair. Some of the coefficients that are most significant in the specification without fixed effects, some lose their significance once fixed effects are added, namely, the bilateral export of investment goods. This is evidence that variation in the cross section of these linkages is significantly associated with changes in transmission, but not within each unit over time. The coefficients on some other linkages, that is, on bilateral investment good imports and bilateral intermediate good imports, increase greatly in size once fixed effects are included. This implies that there is a smaller or more diffuse effect of varying these linkages in the cross section than within each pair over time. In general, there are different mechanisms operating in the cross section than over time, and both sources of variation are informative in different ways.

A few general properties of the point estimates deserve special mention. First, on average, increasing trade linkages by 1% of GDP is associated with stronger transmission than increasing financial linkages by the same amount of GDP. Second, the association is especially strong with the imports of investment and intermediate goods. Within financial assets, larger FDI holdings are associated with decreases in transmission. This does not necessarily imply that trade accounts
for a greater share of the observed change in transmission over the sample, since the changes in financial linkages were much larger than the changes in trade linkages.

Table 4: Estimates of association between linkages and pairwise correlations of output and linkage VAR residuals. (1): simple linkage set. (2): full linkage set. (3): with time trend. (4): without country-pair fixed effects. ***: $p < 0.01$. **: $p < 0.05$. *: $p < 0.1$.

Table (4) revisits the results from Table (2) on the associations between linkages and output correlations, and compares those with the associations between linkages and correlations of the regression residuals $\epsilon$. The first two columns are reproduced from Table (2), and the last two columns are the equivalents for the interquarterly transmission captured in the correlation of the $\epsilon$. As in Table (2), the introduction of fixed effects dramatically shifts the scope of the variation being exploited. Focusing on the last column, which explains changes intraquarterly transmission within country pairs over time, and comparing it with the second column in Table (3), allows us to best distinguish one of the central stylized facts of this paper, that the determinants of correlation of contemporaneous effects are different from those of transmission at one lag.

Among the types of goods traded, a few results stand out. First, in line with existing results in the comovement literature, trade of all types of goods is significantly associated with changes in transmission within each quarter. Second, while it is not possible to distinguish the role of directed imports versus exports within the quarter, when looking at transmission at a lag of one quarter, it is only imports of goods, not exports, that are significantly associated with increased transmission. Third, while increase combined trade in consumption goods is associated with greater unconditional output correlation, it is not significantly associated with any intraquarterly transmission.

Interesting distinctions are also found among the types of assets held. All types of assets are significantly associated with unconditional output correlation, but differences emerge between types of assets in terms of the time horizon of transmission. Namely, portfolio equity is significantly associated with intraquarterly transmission but not transmission at a lag of one quarter, while
Figure 1: The global autoregressive matrix under 1996 linkages as estimated by the linkage VAR. Columns are source countries and columns are partner countries. The lighter and more yellow the shade, the stronger the linkage. The bright columns 11, 12, 19, 20, 34, and 35 represent France, Germany, Italy, Japan, the United Kingdom, and the United States, respectively.

FDI is associated with transmission at both time horizons.

2.5 Propagation within the global network

From the linkage VAR estimates, it is possible to construct an implied autoregressive matrix for the vector of country output growth rates. Let $\hat{A}_t$ be the autoregressive matrix constructed in the following way:

$$\hat{A}_{i,i,t} = \hat{\alpha}_{i,i,0}$$  \hspace{1cm} (15)

$$\hat{A}_{i,j,t} = \hat{\alpha}_{i,j,0} + \sum_{k=1}^{K} \hat{a}_{k} \hat{z}_{i,j,k,t}$$  \hspace{1cm} (16)

Figure 1 displays a representation of the elements of $\hat{A}$, for the specification without country-
pair FE (that is, $\hat{\alpha}_{i,j,0} = 0$), and for $t = 1996$. Brighter and more yellow-hued colors indicate larger elements of the matrix, and therefore stronger links. The matrix is oriented such that shocks originate in column countries and propagate to row countries. A few observations stand out from this figure. First, the diagonal has a noticeable number of bright elements. This indicates that many of countries in this sample have experienced fluctuations that are internally persistent. Second, there are several columns that are brighter and more yellow. These columns represent countries whose cycles are important for a large number of partner countries. Unsurprisingly, these countries are large and rich. In alphabetic order, they are France, Germany, Italy, Japan, the United Kingdom, and the United States.

One of the main contributions of the linkage VAR is to show how this network is associated with trade and financial linkages. From this estimated network we could calculate statistics that have gained increasing interest in the application of network theory to economics, and then use the linkage VAR to tell us how those network statistics are associated with trade and financial linkages. For a given network, changing linkages between a pair of countries changes the edge between those countries, which alters the total global propagation of shocks, depending on the structure of the network. Future work will use the linkage VAR to identify which country-pair linkages are crucial linkages for the global network, and examine how granularity in the structure of the network is related to specific trade or financial linkages.

### 2.6 Global average pairwise correlation

What has driven the changes in average pairwise correlation observed over time? Table 5 reports the change in average pairwise correlation between 1996 and 2010 output growth, residuals from the linkage VAR, and two counterfactuals. Each correlation is calculated at a window of 32 quarters, with the observation at 1996 representing the first 32 quarters of the data and the observation at 2010 representing the last 32 quarters of the data. On average, output growth rates have become much more correlated across countries, with correlation coefficients increasing from 0.14 to 0.48. The average correlation of the estimated linkage VAR residuals $\hat{\epsilon}$ also changes markedly between the first and last 32 quarters of the sample, from 0.09 to 0.30.

How much of the observed change in average output growth correlation between 1996 and 2010 can be attributed to changes in the correlation $\hat{\epsilon}$, and how much to changes in lagged transmission? Recall the formula for the covariance of output growth in a VAR: $\vec{\Sigma}_y = (I - (A \otimes A))^{-1} \vec{\Sigma}_\epsilon$. In the linkage VAR, the matrix $A$ changes over time. Let $\Sigma_{y,t}$ denote the rolling covariance of output growth at time $t \in \{1996, 2010\}$, defined above, and let $\Sigma_{\epsilon,t}$ be defined similarly for the residuals $\hat{\epsilon}$. The third line in Table 5 reports the average output growth correlation for a counterfactual where the covariance of $\hat{\epsilon}$ is fixed to $\Sigma_{\epsilon,1996}$ and $A_t$ varies between the estimated values in 1996 and 2010. This line shows an increase in the average correlation of this synthetic data from 0.012 to 0.33. The fourth line in Table 5 reports the average output growth correlation
for a counterfactual where the autoregressive matrix is fixed at the estimated value in $A_{1996}$ and $\Sigma_{\varepsilon,t}$ varies between the values estimated in 1996 and 2010. This line shows a much smaller increase in average correlation, from 0.12 to 0.13, an increase which is not statistically significant. This is not an exact decomposition of the average pairwise correlation, but it does shed some light on the magnitude of the contributions of each source of variation. The figures in the table suggest that the mechanisms proposed in the literature to account for correlation in pairwise output may not account for the stylized facts in *interquarterly* transmission, since observed correlation seems to be more driven by *intraquarterly* transmission.

<table>
<thead>
<tr>
<th></th>
<th>$t = 1996$</th>
<th>$t = 2010$</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\Sigma}_{y,t}$</td>
<td>0.14</td>
<td>0.48</td>
<td>0.34</td>
</tr>
<tr>
<td>$\hat{\Sigma}_{\varepsilon,t}$</td>
<td>0.09</td>
<td>0.30</td>
<td>0.20</td>
</tr>
<tr>
<td>$A = \hat{A}<em>{1996}$, $\Sigma</em>{\varepsilon} = \hat{\Sigma}_{\varepsilon,t}$</td>
<td>0.12</td>
<td>0.33</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(0.09,0.26)</td>
<td>(0.27,0.52)</td>
<td>(0.18,0.26)</td>
</tr>
<tr>
<td>$A = \hat{A}<em>t$, $\Sigma</em>{\varepsilon} = \hat{\Sigma}_{\varepsilon,1996}$</td>
<td>0.12</td>
<td>0.13</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.09,0.26)</td>
<td>(0.08,0.34)</td>
<td>(-0.07,0.12)</td>
</tr>
</tbody>
</table>

Table 5: Decomposing changes in average bilateral correlation. Bootstrapped 90% confidence intervals (with medians) reflect estimation uncertainty over autoregressive matrices.

### 2.7 Dynamic experiments

Even if lagged transmission is not important for explaining the observed change in global pairwise correlation, it does matter for conditional forecasts of one country’s responses to another country’s shocks. This section shows that the change in the dynamic propagation of shocks can be qualitatively significant.

Using the linkage VAR estimates for the specification without country-pair fixed effects, I conduct a series of experiments where I find the change in propagation implied by a set of changes in bilateral linkages. That is, I construct counterfactual linkages, and from them counterfactual autoregressive matrices for a world VAR, and then report the difference between impulse responses in this counterfactual world and impulse responses under linkages held constant at their 2010 values. However, the global nature of the estimation complicates this exercise. No pair of countries in my sample exists in autarky with respect to the rest of the world. Every change in propagation will have global feedback effects. This means that the change in propagation from one country to another depends on how those two countries interact with the rest of the world. As an illustration, I conduct a range of experiments repeated for two pairs of countries with initial relationships between each other and the rest of the world that vary in the data. The two pairs of countries are 1) two countries that are tightly linked (US, Mexico) and 2) two countries that are distant in linkages and geography (Japan, Brazil).

In each of these experiments, there a 1% shock to the output growth of Foreign (the US nonstructural, and assumed to be uncorrelated with any other shock in the world; a forecast error
Figure 2: 1% shock to Foreign output. Response of Home output (“Foreign → Home”), using linkages from 2010. Dark gray: bootstrapped 80% confidence interval. Light gray: bootstrapped 90% confidence interval.

and Japan, respectively), and the object of interest is the response of Home (Mexico and Brazil, respectively). Figure 2 shows the baseline response of Home, where the impulse responses are constructed from the transition matrix implied by the low-frequency trends that prevailed in 2010 under the linkage VAR without low-frequency fixed effects.

Figures 3 through 5 show the changes in Home’s response relative to the baseline when the counterfactual low-frequency linkage is the linkage in the 2010 data plus 1% of Home GDP. For example, the first panel of Figure 3 plots the change in the response to Mexican GDP from a 1% shock to US GDP when Mexican exports of capital goods to the US are larger by one percent of Mexican GDP than they were in 2010 and US exports of capital goods to Mexico are larger by one percent of US GDP than they were in 2010. The six experiments in Figures 4 through 5 show a balanced-trade increase the bilateral trade of each type of good in turn, followed by a symmetric increase in the bilateral holds of each asset type. The general pattern of response magnitudes parallels the magnitudes and confidence intervals of the point estimates in Table 3 for the specifications without country-pair fixed effects. That is, investment goods, intermediate goods, and FDI have the largest and most significant effects.

These experiments, together with the new stylized facts gleaned from the regression coefficients, represent the empirical contribution of this paper: to extend the usual stylized facts relating linkages and comovement to a setting where what is important is the dynamic propagation of

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16This exercise motivates my use of country-demeaned output growth as my high-frequency variable. Since I am interested in nesting a standard VAR as a special case, I need the high-frequency data to be jointly stationary. If I use HP-filtered real output as my high-frequency variable, I find that the system contains an eigenvalue whose absolute value is close to 1. In some scenarios with counterfactual linkages, the eigenvalues of counterfactual autoregressive matrix for HP-filtered series can exceed 1 in absolute value. When using growth rates instead of HP-filtered output, the counterfactuals remain safely non-explosive.
Figure 3: 1% shock to Foreign output. Change in Response of Home output (“Foreign → Home”) following 1% increase in bilateral low-frequency variables from 2010 baseline. Dark gray: bootstrapped 80% confidence interval. Light gray: bootstrapped 90% confidence interval.
Figure 4: 1% shock to Foreign output. Change in Response of Home output (“Foreign → Home”) following 1% increase in bilateral low-frequency variables from 2010 baseline. Dark gray: bootstrapped 80% confidence interval. Light gray: bootstrapped 90% confidence interval.
Figure 5: 1% shock to Foreign output. Change in Response of Home output (“Foreign → Home”) following 1% increase in bilateral low-frequency variables from 2010 baseline. Dark gray: bootstrapped 80% confidence interval. Light gray: bootstrapped 90% confidence interval.
shocks. The regression results show that different linkages are important for different time horizons,
and the impulse responses illustrate the consequences that these results could have for pairs of
countries over the span of the business cycle.

3 Challenges for international business cycle models

What challenges do the new stylized facts pose for international real business cycle models? In this section, I construct a standard model of international production, trade, and fluctuations. I use this model to show that two mechanisms that have been called upon in the literature to resolve the “trade-comovement” puzzle do not account for the entirety of the new stylized facts, leaving what may be termed the “trade-transmission” puzzle.

In this section, I construct a multi-country international RBC model with two small open economies, and calculate the coefficient from a linkage VAR implied by that model. I find that the model fails to exhibit the same relationship between low-frequency linkages and the dynamic propagation of shocks as found in actual data.

3.1 Model Specification

I follow the literature (e.g. Heathcote and Perri (2002), Kose and Yi (2006)) and focus on total trade between two countries. I abstract from the distinction between types of traded goods and network effects between multiple countries. Since I find that even the most basic relationship between trade linkages and lagged transmission fails to appear in the baseline model, this simpler model suffices. Recent work, such as Johnson (2014), has distinguished between the existence of a trade-comovement puzzle in gross output from that in value added; I make a similar distinction in this model. A model-based evaluation of transmission associated with a full network structure among multiple countries or multiple kinds of goods or trade in various types of assets are meaningful extensions in light of the empirical results, but I leave them for future work.

Production

There are two small open economies, indexed by $i$ and $j$, that face symmetric production structures and household problems. In country $i$, competitive firms produce country-specific good $Q_{it}$, whose price in terms of a globally-traded intermediate good is $P_{it}^Q$. This firm hires labor $L_{it}$ at wage $W_{it}$ and rents a stock of capital $K_{it}$ at rental rate $R_{it}$ to produce value added $V_{it}$. It then combines its value added $V_{it}$ with a composite $M_{it}$ of intermediate goods, for which it pays price
The profit maximization problem of a firm producing $Q_{it}$ is:

$$\max_{Q_{it}, L_{it}, K_{it}} [P_{it}^Q Q_{it} - W_{it} L_{it} - R_{it} K_{it} - P_{it}^M M_{it}]$$  \hspace{1cm} (17)

s.t. $Q_{it} = \left( \theta^{\frac{1}{\sigma}} V_{it}^{\frac{\sigma-1}{\sigma}} + (1 - \theta)^{\frac{1}{\sigma}} M_{it}^{\frac{\sigma-1}{\sigma}} \right) \left( \frac{\sigma}{\sigma-1} \right)$  \hspace{1cm} (18)

and $V_{it} = Z_{it} K_{it}^{\alpha} L_{it}^{1-\alpha}$  \hspace{1cm} (19)

In country $i$, competitive firms produce the composite intermediate good $M_{it}$ by aggregating intermediates from three sources: country $i$ ($X_{iit}$), country $j$ ($X_{jit}$), and the global numeraire ($X_{ROWit}$). The firm’s technology is constant returns to scale ($\omega_s = 1 - \omega_d - \omega_{ROW}$) and its maximization problem is:

$$\max_{Q_{it}, L_{it}, K_{it}} [P_{it}^M M_{it} - P_{it}^Q X_{iit} - P_{jt}^Q X_{jit} - X_{ROWit}]$$  \hspace{1cm} (20)

s.t. $Q_{it} = \left( \omega_s^{\frac{1}{\sigma}} X_{iit}^{\frac{\sigma-1}{\sigma}} + \omega_d^{\frac{1}{\sigma}} X_{jit}^{\frac{\sigma-1}{\sigma}} + \omega_{ROW}^{\frac{1}{\sigma}} X_{ROWit}^{\frac{\sigma-1}{\sigma}} \right) \left( \frac{\sigma}{\sigma-1} \right)$  \hspace{1cm} (21)

**Households**

In country $i$ there is a representative household. The household supplies labor to domestic firms and buys the country-specific good $Q_{it}$ for investment and consumption. The household uses the investment good $I_{it}$ to augment the stock of capital $K_{it}$ it maintains, subject to quadratic adjustment costs. It lends to domestic firms, and that capital depreciates at rate $\delta$. The household also holds a stock of one-period globally traded bonds $B_{it}$ denominated in terms of the global numeraire and earning the constant global return $R^B$. To ensure stationarity, the household pays quadratic adjustment costs on its holding of bonds. Finally, the household receives $\Pi_{it}$, the total profits of all domestic firms, as a lump-sum transfer. Its discount factor is $\beta$ and its marginal utility of consumption is $\lambda_{it}$.

Household $i$ maximizes its discounted utility subject to its budget constraint:

$$\max_{C_{it}, I_{it}, L_{it}, B_{it+1}, K_{it+1}} \sum_{t=0}^{\infty} \beta^t \left( C_{it}^{1-\zeta} - \frac{L_{it+1/\psi}}{1 - \zeta} \right)$$  \hspace{1cm} (22)

s.t. $F^Q C_{it} + F^Q I_{it} + B_{it+1} + \frac{\phi}{2} (B_{it+1} - B)^2 = W_{it} L_{it} + K_{it} R_{it} + B_{it} (1 + R^B) + \Pi_{it}$

$$K_{it+1} = (1 - \delta) K_{it} + \left( 1 - \zeta \left( \frac{I_{it}}{I_{it-1}} - 1 \right) \right) \left( \frac{I_{it}}{I_{it-1}} - 1 \right) I_{it}$$

**Market Clearing**

In equilibrium, the total amount produced of country-specific good $Q_{it}$ will be the sum of the
amounts of consumption, investment and intermediate goods produced for use in both countries.

\[ Q_{it} = C_{it} + I_{it} + X_{iit} + X_{ijt} \]  \hspace{1cm} (22)

**Exogenous processes**

The vector of country-specific productivity, \( Z_t \), follows an autoregressive and potentially cross-correlated process in logs:

\[
\begin{bmatrix}
\log(Z_{it}) \\
\log(Z_{jt})
\end{bmatrix} = \begin{bmatrix} \rho_s & \rho_d \\ \rho_d & \rho_s \end{bmatrix} \begin{bmatrix}
\log(Z_{it-1}) \\
\log(Z_{jt-1})
\end{bmatrix} + \begin{bmatrix} \sigma_s & \sigma_d \\ \sigma_d & \sigma_s \end{bmatrix} \begin{bmatrix}
\epsilon_{it} \\
\epsilon_{jt}
\end{bmatrix} \hspace{1cm} (23)
\]

If \( \sigma_d \neq 0 \), there is contemporaneous correlation of shocks to productivity across countries. If \( \rho_d \neq 0 \), there is lagged transmission of shocks to productivity across countries. Both of these are exogenous sources of comovement.

**3.2 Model Experiments**

I assess this model in light of the new stylized facts by constructing a series of experiments, in each of which I increase trade between the two small open economies by a fixed percentage of steady-state value added and estimate the change in propagation that the model implies from that change in linkages. In each experiment, I calculate what I would obtain from regression the vector of growth rates of value added in each country \( [\Delta \log(V_{it}), \Delta \log(V_{jt})] \) on its lagged counterpart \( [\Delta \log(V_{it-1}), \Delta \log(V_{jt-1})] \). I calculate the coefficients for this VAR for a model where bilateral intermediate good imports are permanently fixed to 1% of domestic intermediate production in each country in steady state; then I calculate the coefficients for this VAR for a model where bilateral intermediate good imports are permanently fixed to 2% of domestic intermediate production in each country in steady state. The two VARs give two autoregressive matrices.\(^{17}\) The difference between the off-diagonal elements of the two autoregressive matrix is the change in international transmission associated with a 1% increase in bilateral imports. This is the linkage VAR coefficient for a specification with country-pair fixed effects. The goal is to assess the strength of the model’s propagation mechanisms relative to those found in the data.

**Calibration.** Since the model is highly stylized, with two small open economies facing the rest of the world, I present a baseline calibration in Table 6 and a range of experiments which describe how sensitive the implied linkage VAR coefficient is to departures from the baseline calibration.

\(^{17}\)In principle, one could estimate a long series of \( [\Delta \log(V_{it}), \Delta \log(V_{jt})] \) and run a VAR on the simulated data. However, since the coefficients I estimate are small, a possible concern might be the estimation error associated with finite-length Monte Carlo simulation of \( [\Delta \log(V_{it}), \Delta \log(V_{jt})] \). To avoid this problem, I obtain a closed-form solution for the autoregressive matrix I would estimate from a Monte Carlo simulation of infinite length, and calculate the linkage VAR coefficient from that solution.
Table 6: Parameter Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of capital in value added</td>
<td>$\alpha$ 0.33</td>
</tr>
<tr>
<td>Share of value added in gross output</td>
<td>$\theta$ 0.5</td>
</tr>
<tr>
<td>Depreciation</td>
<td>$\delta$ 0.025</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$ 0.995</td>
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<tr>
<td>Labor supply elasticity</td>
<td>$\psi$ 0.5</td>
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<tr>
<td>Adjustment cost on bond holdings</td>
<td>$\varphi$ 0.001</td>
</tr>
<tr>
<td>Adjustment cost on investment</td>
<td>$\zeta$ 8</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>$\varsigma$ 1</td>
</tr>
<tr>
<td>Elasticity of substitution between domestic and foreign intermediates</td>
<td>$\phi$ 0.25</td>
</tr>
<tr>
<td>Share of intermediates imported from Rest Of World</td>
<td>$\omega_{ROW}$ 0.1</td>
</tr>
<tr>
<td>TFP autoregressive matrix: lagged domestic TFP</td>
<td>$\rho_s$ 0.8</td>
</tr>
<tr>
<td>TFP autoregressive matrix: lagged foreign TFP</td>
<td>$\rho_d$ 0.199</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<tr>
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<td>0.00046</td>
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</tr>
<tr>
<td>0.00038</td>
<td>0.00044</td>
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</tbody>
</table>

Table 7: Change in lagged transmission in value added growth rates when bilateral imports of intermediates increase from 1% to 2% of total intermediates used in production.

The linkage VAR coefficient estimated from the data is 0.1. That is, a 1% increase in the import of goods is associated with an increase of international transmission (as measured by the off-diagonal element of a VAR matrix) of 0.1. The coefficients reported in Table 7 are three orders of magnitude too small: the model implies a relationship between trade linkages and transmission of business cycles that is much weaker than that in the data. Table 7 also shows that this magnitude is robust to deviations from the benchmark calibration.

Still, variation in some parameters does seem to increase the linkage VAR coefficient: specifically, the linkage VAR coefficient is higher when adjustment costs to bonds are lower, adjustment
costs to investment are higher, foreign and domestic intermediates are tighter complements, intermediate goods make up a larger share of gross output, the intertemporal elasticity of substitution is lower, the labor supply elasticity is higher, and the share of Rest of World goods in imports is higher. Several of these patterns extend the existing literature on trade and comovement to the relationship between trade and transmission at a lag. For example, the role of the elasticity of substitution between domestic and foreign intermediates is in line with Heathcote and Perri (2002), Kose and Yi (2006), and similar studies: tighter complementarity leads to increased comovement.

In contrast, the exogenous processes play a different role for lagged transmission than for comovement in general: Johnson (2014) finds that increasing the contemporaneous correlation of shocks to TFP increases the marginal effect of trade on the correlation of output. Table 7 shows that increasing the contemporaneous correlation of shocks to TFP (achieved by increasing $\sigma_d$, since the contemporaneous correlation of the shocks to TFP is $\frac{2\rho_{ds}}{\sigma_s}$) does not strengthen the relationship between trade and transmission of shocks at a lag. One can interpret the results of Johnson (2014) as a recognition that the international RBC model relies on properties of the exogenous process to explain moments of endogenous variables: to account for the observed international correlation of output, shocks to TFP must themselves be correlated. Table 7 shows that this same shortcoming of the international RBC model holds for transmission at a lag; increasing bilateral trade cannot account for transmission in the model, which means that the model must rely on exogenous transmission of TFP shocks (high $\rho_d$) instead. Moreover, the marginal effect of trade on the transmission of shocks to output is greater in the presence of greater exogenous transmission, as shown above.
Table 8: Change in lagged transmission in gross output growth rates when bilateral imports of intermediates increase from 1% to 2% of total intermediates used in production.

<table>
<thead>
<tr>
<th>( \varphi = 0.1 )</th>
<th>( \varphi = 0.01 )</th>
<th>( \varphi = 0.001 )</th>
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<td>( \zeta = 16 )</td>
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<td>( \phi = 0.5 )</td>
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</tr>
<tr>
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<tr>
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<td>( \varsigma = 1.75 )</td>
<td>( \varsigma = 2 )</td>
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<tr>
<td>( \psi = 2 )</td>
<td>( \psi = 1.5 )</td>
<td>( \psi = 1 )</td>
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<td>( \psi = 0.5 )</td>
</tr>
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<td>( \sigma_d = 0 )</td>
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<tr>
<td>( \omega_{ROW} = 0.05 )</td>
<td>( \omega_{ROW} = 0.075 )</td>
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<td>0.00023</td>
<td>0.00024</td>
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</tr>
</tbody>
</table>

Are the small magnitudes of the linkage VAR coefficient a property of the model that holds only for value added? Table 8 shows that the model-based coefficients are also very small when the linkage VAR is run on the growth rates gross output instead of the growth rates of value added. As with value added in Table 7, this result is invariant to the contemporaneous correlation of shocks to TFP (seen by varying \( \sigma_d \)).

Since the mechanisms which have been proposed to partially resolve the trade-comovement puzzle (increasing the complementarity of domestic and foreign inputs, increasing the contemporaneous cross-country correlation of exogenous shocks) do not substantially reconcile the model’s predictions with the data, we can conclude that the trade-comovement puzzle has two distinct components: the trade-correlation puzzle and the trade-transmission puzzle. Specifically, the relationship between trade and lagged transmission is tiny in the model. This shortcoming is fundamentally different from a failure to account for the relationship between trade and correlation: resolution of the trade-transmission puzzle requires a mechanism that amplifies transmission at a lag in a way that is complementary to trade. In the following section I describe a mechanism that can help strengthen the model’s predicted relationship between trade and transmission.
4 Enhancing delayed transmission with sticky information

In this section, I propose a form of sticky information can enhance mechanisms that give rise to comovement, and help address the “trade-transmission” puzzle. This section does not aim to reproduce all the novel stylized facts described in Section 2. As shown in Section 3, getting any substantial delayed transmission is difficult with standard mechanisms in a IRBC framework. The contribution of this section is to propose a mechanism which can give rise to delayed transmission in gross output, describe how it works in an extremely simple setting, and then explore its quantitative significance in a medium-scale model with both gross output and value added, especially as a complement to existing mechanisms which have been explored in connection with the “trade-comovement” puzzle.

4.1 Simple model

I use a version of the model used for exposition in Johnson (2014) that posits two countries trading country-specific intermediate inputs that are used in production in each country. As in Johnson (2014), I hold both the capital and the labor input fixed to focus on the propagation mechanisms arising from trade in intermediates alone. Holding the capital and labor inputs fixed also isolates the transmission that arises solely in terms of gross output, since there are no endogenous movements in value added.

A representative competitive firm in country $i$ produces output $Q_{it}$ using value added $V_{it}$, domestic intermediate goods $X_{iit}$, and foreign intermediate goods $X_{jit}$\footnote{For intermediate goods $X$, the first index denotes the country where the good is produced and the second index denotes the country where the good is used.}. Value added $V_{it}$ is linear in labor input $L_{it}$, subject to productivity $Z_{it}$. In units of its own production, the firm pays relative price $S_{jt}$ for the foreign intermediate good. The firm takes all factor prices as given. The firm’s profit $\Pi_{it}$ is its revenue less the costs of its inputs.

\[ Q_{it} = (V_{it})^{\theta} (X_{iit}^{\omega} X_{jit}^{1-\omega})^{1-\theta} \]  
\[ \Pi_{it} = Q_{it} - W_{it}L_{it} - X_{iit} - S_{jt}X_{jit} \]  
\[ V_{it} = Z_{it}L_{it} \]

Throughout this section, there will be two alternatives for the firm’s maximization problem. The first alternative supposes that the firm has full information over the factor prices it faces, the technology it uses, and so on. In this case, the firm’s problem is standard. It maximizes its profit

\[ \max_{Q_{it}, X_{iit}, X_{jit}, V_{it}, L_{it}} \Pi_{it} \text{ s.t. } (24), (25), (26) \]
Its optimality conditions are standard. It equates marginal cost to marginal product of each input.

$$\theta Q_{it} = W_{it}L_{it}$$  \hspace{1cm} (28)  

$$(1 - \theta)(1 - \omega)Q_{it} = S_{jt}X_{jit}$$  \hspace{1cm} (29)  

$$(1 - \theta)\omega Q_{it} = X_{iit}$$  \hspace{1cm} (30)  

Because the production function exhibits constant returns to scale, all profits $\Pi_{it}$ are equal to zero in equilibrium.

The second alternative for the firm’s maximization problem supposes that the firm has a very specific and very stark form of sticky information: the firm is fully informed about its productivity and domestic factor prices, but maximizes profit under beliefs about the relative price of the foreign input that were formed in the previous period. Let $E_{[S_{jt}[I_{t-1}]]}$ denote the expectation operator with respect to the relative price $S_{jt}$ under the information set $I_{t-1}$. This is a very simple form of sticky information that is useful to illustrate the main points of this section. To draw an analogy, it is to the sticky information of Mankiw and Reis (2002) as the one-period-stickness of Rotemberg (1982) is to Calvo (1983) pricing. It also bears a resemblance to the informational friction of Dasgupta and Mondria (2014), where importers lack complete information about the prices of the goods they import, due to rational inattention.

$$\max_{Q_{it},X_{iit},X_{jit},V_{it},\Pi_{it}} E_{[S_{jt}[I_{t-1}]]} [\Pi_{it}] \text{ s.t. } (24), (25), (26)$$  \hspace{1cm} (31)  

Under this form of sticky information, it is possible that the firm’s optimality conditions are standard, except for its optimal choice of the foreign input. There, the foreign input is chosen under information about the relative foreign price that is one period out of date.

$$\theta Q_{it} = W_{it}L_{it}$$  \hspace{1cm} (32)  

$$(1 - \theta)(1 - \omega)Q_{it} = E_{t-1}[S_{jt}]X_{jit}$$  \hspace{1cm} (33)  

$$(1 - \theta)\omega Q_{it} = X_{iit}$$  \hspace{1cm} (34)  

$$S_{jt} E_{t-1}[S_{jt}]$$  \hspace{1cm} (35)  

Under sticky information, it is possible that the firm’s profits $\Pi_{it}$ are not equal to zero in equilibrium:

$$\Pi_{it} = Q_{it} - W_{it}L_{it} - X_{iit} - S_{jt}X_{jit}$$  \hspace{1cm} (36)  

$$= Q_{it} - \theta Q_{it} - (1 - \theta)\omega Q_{it} - (1 - \theta)(1 - \omega)Q_{it} \frac{S_{jt}}{E_{t-1}[S_{jt}]}$$  \hspace{1cm} (37)  

$$= (1 - \theta)(1 - \omega)Q_{it} \left(1 - \frac{S_{jt}}{E_{t-1}[S_{jt}]}\right)$$  \hspace{1cm} (38)
Profits are only equal to zero if the realized relative price $S_{jt}$ is equal to its previously expected value. Otherwise, the firm receives nonzero profits. These profits are not competed away in equilibrium because this firm is representative, and all firms in a given country are subject to the same information frictions.

The problems and optimality conditions for firms in country $j$ are symmetric. **Households** provide a single unit of labor to domestic firms and consume some final use $F_{it}$ of domestic production, over which they have log utility. They receive any profits from domestic firms as a lump-sum transfer.

$$\max_{F_{it}} \log(F_{it}) \text{ s.t. } F_{it} = W_{it}L_{it} + \Pi_{it}$$

(39)

(40)

**Market clearing.** In equilibrium, total gross output $Q_{it}$ in country $i$ is equal to the sum of its final use $F_{it}$, its use by domestic firms as an intermediate $X_{iit}$, and its use by foreign firms as an intermediate $X_{ijt}$.

$$Q_{it} = F_{it} + X_{iit} + X_{ijt}$$

(41)

Additionally, the price of country-$j$ goods in terms of country-$i$ goods, $S_{jt}$, is the reciprocal of the price of country-$i$ goods in terms of country-$j$ goods, $S_{it}$: $S_{jt} = 1/S_{it}$.

### 4.2 Transmission in the simple model

The Appendix lists the full collection of equilibrium conditions for country $i$, with country $j$ being symmetric. In a steady state, there are no shocks, and therefore the expectations taken with respect to old information are not mistaken. The steady state is symmetric between the two countries. I consider a log-linear approximation of this equilibrium around the steady-state. After some algebra, the equilibrium reduces to four equations which describe the relationship between log-deviations of output in both countries, log-deviations of the relative prices, and log-deviations of technology. Let $\tilde{\theta} = \frac{1-\theta}{\theta}(1 - \omega)$. This term is larger if either $\theta$ is smaller (the share of intermediates in production is larger) or $\omega$ is smaller (the share of foreign intermediates in production is larger).

$$q_{it} = z_{it} - \tilde{\theta}E_{t-1}[s_{jt}]$$

$$q_{jt} = z_{jt} - \tilde{\theta}E_{t-1}[s_{it}]$$

$$q_{jt} - q_{it} = s_{jt} + E_{t-1}[s_{it} - s_{jt}]$$

$$s_{jt} = -s_{it}$$
A thought experiment will shed light on these conditions. Suppose that at \( t = 0 \), the world economy is at the steady state. In \( t = 1 \), unexpected innovations to the processes for \( z_{it} \) and \( z_{jt} \) arrive. For \( t > 1 \), no future innovations arrive, and \( z_{it} \) and \( z_{jt} \) evolve according to some known law of motion, conditional on the innovations in \( t = 1 \). Since the innovations were unexpected, firms expected in \( t = 0 \) the relative prices to remain at steady state into \( t = 1 \). Therefore \( E_{t-1}[s_{jt}] = E_{t-1}[s_{it}] = 0 \). Remember that firms are always fully aware of their own technology and factor prices, but they have sticky information about the relative price of their imported intermediate input. In \( t = 1 \) of this scenario, firms choose inputs and production as if the relative price of imports has not changed. In log-deviations, production moves one-for-one with domestic technology. Realized relative prices also respond in equilibrium as a matter of market clearing, but they do not influence the optimal choice of production; they only cause firms to reap nonzero profits in \( t = 1 \).

\[
\begin{align*}
q_{i1} &= z_{i1} \\
q_{j1} &= z_{j1} \\
s_{i1} &= z_{i1} - z_{j1} \\
s_{j1} &= z_{j1} - z_{i1}
\end{align*}
\]

For all \( t > 1 \) in this thought experiment, all firms have updated their information sets. Firms see that relative prices have changed, and can infer the innovations that gave rise to the equilibrium allocations in \( t = 1 \). They correctly know the evolution of all relative prices conditional on the realization of the innovations to productivity up to \( t = 1 \). In this thought experiment, there are no further innovations. Therefore, the expectations \( E_{t-1}[s_{jt}] \) and \( E_{t-1}[s_{it}] \) of the firms at every \( t > 1 \) coincide with the realizations of \( s_{jt} \) and \( s_{it} \), respectively. Firms’ import decisions respond to movements in relative prices, since the realized movements of relative prices are in the firms’ information sets. As a consequence, for \( t > 1 \), output in both countries is a function not only of domestic technology, but also of foreign technology.

\[
\begin{align*}
q_{it} &= \frac{1 + \tilde{\theta}}{1 + 2\tilde{\theta}} z_{it} + \frac{\tilde{\theta}}{1 + 2\tilde{\theta}} z_{jt} \\
q_{jt} &= \frac{\tilde{\theta}}{1 + 2\tilde{\theta}} z_{it} + \frac{1 + \tilde{\theta}}{1 + 2\tilde{\theta}} z_{jt} \\
s_{it} &= -\frac{1}{1 + 2\tilde{\theta}} z_{it} + \frac{1}{1 + 2\tilde{\theta}} z_{jt} \\
s_{jt} &= \frac{1}{1 + 2\tilde{\theta}} z_{it} - \frac{1}{1 + 2\tilde{\theta}} z_{jt}
\end{align*}
\]

Figure 4.2 illustrates this in an impulse response to a structural shock. Let \( \omega = 0.5 \) and
\( \theta = 0.5, \) so \( \bar{\theta} = 0.5, \) \( \frac{1+\bar{\theta}}{1+2\bar{\theta}} = 0.75, \) and \( \frac{\bar{\theta}}{1+2\bar{\theta}} = 0.25. \) Suppose that \( z_{it} \) and \( z_{jt} \) follow independent autoregressive processes with persistence 0.75. Suppose there is a shock to \( z_{jt} \) alone in \( t = 1. \) Output in country \( j \) responds to this shock in period 1 regardless of the information assumption on relative prices.\(^{19}\) When firms have full information about relative prices, output in country \( i \) responds in the first period as well. However, when firms have sticky information about relative prices, output in country \( i \) does not respond to the shock in country \( j \) until \( t = 2. \)

![Figure 6: 1% shock to source country productivity. Responses of source and partner country output, under full information and sticky information.](image)

The delayed response to foreign shocks shown in Figure 4.2 is the key to reproducing the stylized facts from the linkage VAR. However, there is an additional step to connect the structural model to the linkage VAR. What is shown in Figure is the response to a structural shock, and the linkage VAR does not identify structural shocks. In order to connect any structural model to the linkage VAR results, it is necessary to perform the same exercise on data simulated from the structural model as was performed on the real-world data. Imagine simulating data on the model above, and running OLS on \((q_{it}, q_{jt})\) to get a autoregressive matrix \( \hat{\beta}_q. \) In this simple model, there’s no need to simulate data, since this statistic can be calculated analytically. For an infinite series of data, the moments of the data needed to estimate OLS would be the unconditional expectations. We can find the unconditional expectations from the policy functions for \( q_t = [q_{it}, q_{jt}]' \) and \( z_t = [z_{it}, z_{jt}]'. \)

The VAR coefficients are a function of unconditional moments of \( q_t: \)

\[
\beta_q = E[q_{t-1}q_{t-1}]^{-1}E[q_{t-1}q_{t}']
\]

In contrast, the contemporary covariance of \( q_t \) is simply

\[
\Sigma_q = E[q_tq_t']
\]

\(^{19}\)The size of the response does depend on informational assumptions.
In what follows, I assume the following laws of motion for productivity, where \( [\epsilon_{z, it-1}, \epsilon_{z, jt-1}]' \sim N(0, I) \):

\[
\begin{bmatrix}
z_{it} \\
z_{jt}
\end{bmatrix}
= 
\begin{bmatrix}
\rho_s & \rho_d \\
\rho_d & \rho_s
\end{bmatrix}
\begin{bmatrix}
z_{it-1} \\
z_{jt-1}
\end{bmatrix}
+ 
\begin{bmatrix}
\sigma_s & \sigma_d \\
\sigma_d & \sigma_s
\end{bmatrix}
\begin{bmatrix}
\epsilon_{z, it-1} \\
\epsilon_{z, jt-1}
\end{bmatrix}
\]

This specification allows a flexible range of intra- and inter-quarterly *exogenous* transmission. If \( \sigma_d \neq 0 \), then the innovations to productivity in each country are correlated. In other words, there is intra-quarter exogenous transmission of productivity shocks. If \( \rho_s > 0 \), then there is persistence of the productivity process within each country. That is, once productivity is shocked, regardless of the origin of the shock, the effect of the shock will be persistent as long as \( \rho_s > 0 \). When the final parameter, \( \rho_d \), is different from zero, it represents the presence of inter-quarterly exogenous transmission. For example, consider a case where \( \sigma_d = 0 \) but \( \rho_d > 0 \). Then innovations abroad will have a contemporaneous effect on foreign productivity but not home productivity. However, that same shock will have an effect on home productivity with a lag of one quarter.

One can calculate an analytic expression for the unconditional correlation of \( z_t \) and find that it is a function of \( (\rho_s, \rho_d, \sigma_s, \sigma_d) \). Similarly, estimating OLS on a time series for \( z_t \) with infinite data would yield \( \beta_z = \left[ \begin{array}{cc} \rho_s & \rho_d \\ \rho_d & \rho_s \end{array} \right] \). But the stylized facts from the estimated linkage VAR are not concerning exogenous productivity; they are facts about the estimated relationships between endogenous variables. In this structural model, it is possible to distinguish exogenous transmission of productivity to potentially asymmetric endogenous responses to those shocks. Propositions 1 and 2 use the analytic expressions for the equilibrium policy function for \( q_t \) as a function of \( z_{t-1} \) and \( \epsilon_t \) to perform comparative statics on \( \Sigma_q \) and \( \beta_q \). The main goal of the Propositions is to distinguish intra- from inter-quarterly transmission. Namely, several mechanisms that relate trade intensity and unconditional correlation act to increase intra-quarterly transmission without increasing inter-quarterly transmission.

First, consider \( \Sigma_q \). To test whether trade leads to greater unconditional correlation, it suffices to consider the off-diagonal element of this matrix.

**Proposition 1** As \( \tilde{\theta} \) increases, the off-diagonal elements of \( \Sigma_q \) increase under both flexible and sticky information. Additionally, if \( \sigma_s > \sigma_d \), the off-diagonal elements of \( \Sigma_q \) are also increasing functions of \( \sigma_d \).

**Proof.** See Appendix

Proposition 1 says that more trade (larger \( \tilde{\theta} \)) leads to greater covariance between the two countries’ output\(^{20}\) independent of information structure. This result parallels results in Johnson (2014). Also as in Johnson (2014), increasing the *exogenous* correlation of innovations to produc-

---

\(^{20}\)Propositions 1 and 2 are stated in terms moments of the log-levels of output, but it is easy to show that they hold quantitatively for first-differences in log output (that is, output growth rates) as well.
tivity increases the *endogenous* correlation of output, under the assumption that the initial impact of a domestic shock is greater at home than abroad.

In contrast, consider $\beta_q$, the coefficients that would be estimated in a VAR on output.

**Proposition 2** Under sticky information, as long as $\rho_s > \rho_d$, the off-diagonal elements of $\beta_q$ increase as $\tilde{\theta}$ increases. Under full information, the off-diagonal elements of $\beta_q$ are independent of $\tilde{\theta}$. Additionally, the off-diagonal elements of $\beta_q$ are independent of both $\sigma_s$ and $\sigma_d$, under both sticky and flexible information. Finally, the off-diagonal elements of $\beta_q$ are increasing in $\rho_d$.

**Proof.** See Appendix

Proposition 2 shows that parameters that increase the unconditional correlation of output in equilibrium do not necessarily increase the off-diagonal elements of the VAR matrix. Namely, increasing the contemporaneous correlation of the innovations to technology, while increasing the unconditional correlation of output, does not increase lagged endogenous transmission. The parameter that does increase lagged endogenous transmission is $\rho_d$, the parameter that governs lagged exogenous transmission. As Johnson (2014) shows, increasing exogenous comovement can help boost measured endogenous comovement. But the relevant parameter in this application, where lagged transmission is the statistic of interest, is not $\sigma_d$, but $\rho_d$.

Most important is the comparative static on $\tilde{\theta}$. To reproduce the results from the linkage VAR, increasing this parameter needs to lead to greater lagged transmission. As discussed in the section 2, the estimated linkage VAR shows that trade of intermediate goods is associated with lagged transmission from abroad that is orthogonal to the domestic persistence of contemporaneous transmission from abroad. Proposition 2 shows that, in this simple model (and under the additional standard assumption of autoregressive exogenous processes), there is no lagged international transmission that is orthogonal to the domestic persistence of previous contemporaneous international transmission, unless information about international relative prices is sticky. The way sticky information achieves this result is by delaying the initial impact of foreign shocks at home by one period. Finally, this result holds even if there is no exogenous lagged transmission (that is, if $\rho_d = 0$).

In this section, I have given a very stylized model to demonstrate the challenges of rationalizing the stylized facts found in the linkage VAR. I have shown that the challenge to replicate these facts is greater than the challenge of the trade-comovement puzzle in the literature; increasing the correlation of the innovations to productivity does not deliver lagged endogenous transmission. In fact, in the full-information version of this stylized model, trade intensity has no effect on lagged endogenous transmission at all, even in the presence of lagged exogenous transmission. I have also proposed a stark form of sticky information that does deliver lagged endogenous transmission, and furthermore a degree of lagged endogenous transmission that is increasing in bilateral trade intensity.
4.3 Revisiting full model with sticky information

To assess the quantitative impact of sticky information, I introduce it into the larger model of the previous section. For simplicity, I assume that all firms who import goods have one-period-ahead sticky information. That is, a firm in country $i$ maximizes not profits $\Pi_{it}$, but instead profits under sticky information $E_{[\Pi_{it}^s | I_{i,t-1}]}[\Pi_{it}]$. One could assume that only a fraction of agents have sticky information, but the assumption taken here is the starkest, for simplicity.

Table 9 repeats the experiments of Table 7 under the assumption of sticky information. These coefficients are the changes in the off-diagonal element of an autoregressive matrix estimated in a VAR of the two small open economies' growth rates of value added, comparing the transmission under a world where foreign intermediates make up 1% of total intermediates ($\omega_d = 0.01$) with a world where foreign intermediates make up 2% of total intermediates ($\omega_d = 0.02$). The empirically estimated coefficient is 0.1. In the absence of sticky information, these coefficients differ from that in the data by several orders of magnitude; with sticky information, the coefficients are roughly 10 times larger, increasing from 0.5% of the empirical linkage VAR estimate to 5% of the empirical estimate. In other words, the coefficients in Table 9 are still small relative to the data, but are less small under than full information. The presence of sticky information robustly improves upon full information for all deviations from the baseline parameter calibration, with the exception of the case where foreign and domestic intermediates are substantially more substitutable than Cobb-Douglas ($\phi = 1.5$).
<table>
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Table 9: Change in lagged transmission in value added growth rates when bilateral imports of intermediates increase from 1% to 2% of total intermediates used in production, under sticky information. Also indicated are difference over, and ratio to coefficient of lagged transmission under full information.
Table 10: Change in lagged transmission in gross output growth rates when bilateral imports of intermediates increase from 1% to 2% of total intermediates used in production, under sticky information. Also indicated are difference over, and ratio to coefficient of lagged transmission under full information.

Table 10 shows that sticky information amplifies the increase in lagged transmission in gross output even more than value added; the the model-implied linkage VAR coefficients under sticky information range from 30 to almost 200 times those under full information. Again, both the
coefficients and the effect of sticky information are invariant to the contemporaneous correlation of the shocks, further distinguishing the contrast trade-transmission puzzle from the trade-correlation puzzle. This result parallels the quantitative findings of [Johnson (2014)], who found that in a similar model (and full information), gross output comoves more strongly than real value added, due to comovement in input use. Table 10 gives evidence that sticky information amplifies the lagged transmission of shocks through trade more so for gross output than value added, largely through similar channels in input use.

In summary, the results of the previous two sections show that a standard model of international real business cycles faces serious shortcomings in reproducing the empirical results from the linkage VAR. This amounts to a new twist on an old puzzle: “trade-transmission” as distinct from “trade-correlation.” Finally, sticky information is a mechanism that can help move the model a bit closer to the data in a way complementary to the insights of the literature; since it serves to enhance lagged transmission under a variety of parameter values, it is plausible that informational frictions might play a significant role in a more complete resolution of the “trade-transmission” puzzle.

5 Conclusions

This paper has attempted to answer the question of how international business cycles vary systematically across countries and country pairs according to low-frequency trends such as trade flows and asset holdings. The “linkage VAR” estimated in this paper brings together previous literatures on the determinants of comovement and international or panel VARs. In explaining propagation between countries’ output alone, trade and financial linkages are associated with transmission of shocks at a lag of one quarter, import and export intensity are asymmetrically associated with shock transmission, and the import of goods used for capital formation has the largest and most significant association with transmission of shocks. In addition, this paper has shown that a reasonably standard international real business cycle model cannot replicate these facts. Even under a number of parameter values for the model, the relationship between trade and lagged transmission in the model is tiny. This shortcoming, while related to the trade-correlation puzzle, requires a fundamentally different mechanism to resolve it: what is needed is a mechanism that amplifies transmission at a lag in a way that is complementary to trade. I show that the same model, augmented with sticky information on the part of importers, can help overcome some of the gap between the model and the data: sticky information increases the trade-transmission relationship by a factor of 10 in the baseline calibration, and with even stronger results for transmission of innovations to gross output. However, while this paper has refined the “trade-comovement” puzzle into the “trade-correlation” and “trade-transmission” puzzles, both are still alive and well.
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Appendix (for online publication)

A.1 Equilibrium conditions and policy functions for simple model

Summary of equilibrium conditions for country $i$ under sticky information. For full information, replace $E_{t-1}[S_{jt}]$ with $S_{jt}$.

\[ Q_{it} = (V_{it})^\theta (X_{iit}^{\omega} X_{jit}^{1-\omega})^{1-\theta} \]
\[ Q_{it} = F_{it} + X_{iit} + X_{ijt} \]
\[ \theta Q_{it} = W_{it} L_{it} \]
\[ (1-\theta)(1-\omega)Q_{it} = E_{t-1}[S_{jt}]X_{jit} \]
\[ (1-\theta)\omega Q_{it} = X_{iit} \]
\[ \Pi_{it} = Q_{it} - W_{it} L_{it} - X_{iit} - S_{jt} X_{jit} \]
\[ W_{it} V_{it} + \Pi_{it} = F_{it} \]
\[ V_{it} = Z_{it} L_{it} \]

Symmetric Steady State

\[ Q = ((1-\theta)(\omega)^{\omega}(1-\omega)^{1-\omega})^{(1-\theta)/\theta} \]
\[ \theta Q = F \]
\[ (1-\theta)(1-\omega)Q = X_d \]
\[ (1-\theta)\omega Q = X_s \]
\[ V = 1 \]
\[ Z = 1 \]
\[ S = 1 \]
\[ L = 1 \]

Log-linearization

\[ q_{it} = \theta z_{it} + (1-\theta) (\omega x_{iit} + (1-\omega)x_{jit}) \]
\[ q_{it} = f_{it}\theta + x_{iit}(1-\theta)\omega + x_{ijt}(1-\theta)(1-\omega) \]
\[ q_{it} = E_{t-1}[s_{jt}] + x_{jit} \]
\[ q_{it} = x_{iit} \]
\[ q_{it} = f_{it}\theta + x_{iit}\omega(1-\theta) + (s_{jt} + x_{jit})(1-\omega)(1-\theta) \]

Solve for general policy functions:

\[ q_t = q_z z_{t-1} + q_e \epsilon_t \]
\[ z_t = z_z z_{t-1} + z_e \epsilon_t \]
\[ q_{it} = q_{i,z} z_{it-1} + q_{i,e} \epsilon_{z,it} + q_{i,e} \epsilon_{z,jt} + q_{i,e} \epsilon_{z,jt} \]
\[ q_{jt} = q_{j,z} z_{jt-1} + q_{j,e} \epsilon_{z,it} + q_{j,e} \epsilon_{z,jt} + q_{j,e} \epsilon_{z,jt} \]
\[ s_{it} = s_{i,z} z_{it-1} + s_{i,e} \epsilon_{z,it} + s_{i,e} \epsilon_{z,jt} \]
\[ s_{jt} = s_{j,z} z_{jt-1} + s_{j,e} \epsilon_{z,it} + s_{j,e} \epsilon_{z,jt} \]
\[ E_{t-1}[s_{it}] = s_{i,z} z_{it-1} + s_{i,e} \epsilon_{z,jt} \]
\[ E_{t-1}[s_{jt}] = s_{j,z} z_{jt-1} + s_{j,e} \epsilon_{z,jt} \]

Policy functions, solved in the presence of sticky information:

\[ q_{i,z} = q_{j,z} = \frac{1 + \theta}{1 + 2\theta} \rho_s + \frac{\tilde{\theta}}{1 + 2\theta} \rho_d \]
\[ q_{i,z} = q_{j,z} = \frac{1 + \theta}{1 + 2\theta} \rho_d + \frac{\tilde{\theta}}{1 + 2\theta} \rho_s \]
\[ s_{i,z} = s_{j,z} = \frac{1}{1 + 2\theta} \rho_d - \frac{1}{1 + 2\theta} \rho_s \]
\[ s_{i,z} = s_{j,z} = \frac{1}{1 + 2\theta} \rho_s - \frac{1}{1 + 2\theta} \rho_d \]
\[ q_{i,e} = q_{j,e} = \sigma_s \]
\[ q_{i,e} = q_{j,e} = \sigma_d \]
\[ s_{i,e} = s_{j,e} = \sigma_s - \sigma_d \]
\[ s_{i,e} = s_{j,e} = \sigma_d - \sigma_s \]

Policy functions, solved in the absence of sticky information. They are the same coefficients on lagged \( z \), but different on the \( \epsilon \). In fact, the policy functions for the \( \epsilon \) look just like those on the lagged \( z \):

\[ q_{i,z} = q_{j,z} = \frac{1 + \theta}{1 + 2\theta} \rho_s + \frac{\tilde{\theta}}{1 + 2\theta} \rho_d \]
\[ q_{i,z} = q_{j,z} = \frac{1 + \theta}{1 + 2\theta} \rho_d + \frac{\tilde{\theta}}{1 + 2\theta} \rho_s \]
\[ s_{i,z} = s_{j,z} = \frac{1}{1 + 2\theta} \rho_s - \frac{1}{1 + 2\theta} \rho_d \]
\[ s_{i,z} = s_{j,z} = \frac{1}{1 + 2\theta} \rho_d - \frac{1}{1 + 2\theta} \rho_s \]
\[ q_{i,e} = q_{j,e} = \frac{1 + \theta}{1 + 2\theta} \sigma_s + \frac{\tilde{\theta}}{1 + 2\theta} \sigma_d \]
\[ q_{i,e} = q_{j,e} = \frac{1 + \theta}{1 + 2\theta} \sigma_d + \frac{\tilde{\theta}}{1 + 2\theta} \sigma_s \]
\[ s_{i,e} = s_{j,e} = \frac{1}{1 + 2\theta} \sigma_s - \frac{1}{1 + 2\theta} \sigma_d \]
\[ s_{i,e} = s_{j,e} = \frac{1}{1 + 2\theta} \sigma_d - \frac{1}{1 + 2\theta} \sigma_s \]

The goal, which is a function of unconditional moments of \( q_t \):

\[ \hat{\beta} = E[q_t q_t']^{-1} E[q_{t-1} q_t] \]
First, find unconditional moments of $z_t$:

$$E[z_t z_t'] = z_t E[z_t z_t'] z_t + z_e z_e'$$

$$\Rightarrow \text{vec}(E[z_t z_t']) = (I - (z_e \otimes z_e))^{-1} \text{vec}(z_e z_e')$$

$$E[z_t-1 z_t'] = E[z_t z_t'] z_t'$$

Now, use those to find unconditional moments of $q_t$:

$$\Sigma_q = E[q_t q_t'] = q_t E[z_t z_t'] q_t' + q_e q_e'$$

$$E[q_{t-1} q_t'] = q_t z_t z_t' q_t'$$

This yields the OLS estimate of interest:

$$\hat{\beta} = (q_t(I - (z_e \otimes z_e))^{-1} \text{vec}(z_e z_e') q_t' + q_e q_e')^{-1} q_t(I - (z_e \otimes z_e))^{-1} \text{vec}(z_e z_e') z_t' q_t'$$

**Proposition 1** First, assume that the process for productivity is stationary: $|\rho_d + \rho_s| < 1$ and $|\rho_d - \rho_s| < 1$. Let $\Sigma_{q,d}$ be an off-diagonal element of $\Sigma_q$. Since the two countries are symmetric, both off-diagonal elements are the same.

Some algebra shows that $\frac{d\Sigma_q}{d\theta} = \frac{2(\sigma_d - \sigma_s)^2}{(1 - (\rho_d - \rho_s)^2)(1 + \theta)^2}$ under full information and $\frac{d\Sigma_q}{d\theta} = \frac{2(\sigma_d - \sigma_s)^2(\rho_d - \rho_s)^2}{(1 - (\rho_d - \rho_s)^2)(1 + \theta)^2}$ under sticky information. Both quantities are positive.

Similarly, $\frac{d\Sigma_{q,d}}{d\sigma_d} = \frac{\sigma_s - \sigma_d}{(1 + \theta)^2} + \frac{\sigma_d + \sigma_s}{(1 - (\rho_d + \rho_s)^2)}$ under full information, and $\frac{d\Sigma_{q,d}}{d\sigma_d} = \frac{2(\sigma_s - \sigma_d)(\rho_d - \rho_s)^2}{(1 - (\rho_d - \rho_s)^2)(1 + \theta)(1 - (\rho_d + \rho_s)^2)(2 - (\rho_d + \rho_s)^2)}$ under sticky information. Both quantities are positive as long as $\sigma_s > \sigma_d$.

**Proposition 2** Let $\beta_{q,d}$ be an off-diagonal element of $\beta_q$. Since the two countries are symmetric, both off-diagonal elements are the same. Some algebra shows that $\beta_{q,d} = \rho_d^3 + 3\rho_d \rho_s^2$ under full information. This quantity is not a function of $\sigma_s$, $\sigma_d$, or $\hat{\theta}$. It is increasing in $\rho_d$.

Some algebra shows that $\beta_{q,d} = \frac{1}{2} \left( (\rho_d + \rho_s)^3 + (\rho_d - \rho_s)^3 \right)$ under sticky information. This quantity is not a function of $\sigma_s$ or $\sigma_d$. $\frac{d\beta_{q,d}}{d\rho_d} = \frac{2(1 - (\rho_d - \rho_s)^2)(\rho_s - \rho_d)^2}{(1 + 4(1 - (\rho_d - \rho_s)^2)(1 + \theta)^2)}$, which is positive as long as $\rho_s > \rho_d$. 

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A.2 Equilibrium conditions of full model with sticky information

\[
V_{it} = Z_{it} K_{it} \alpha_{it} I_{it}^{1-\alpha} \\
V_{jt} = Z_{jt} K_{jt} \alpha_{jt} I_{jt}^{1-\alpha} \\
P_{it}^{Q} = \left( \theta(P_{it}^{V})^{1-\alpha} + (1 - \theta)(P_{jt}^{M})^{1-\alpha} \right)^{1/\gamma} \\
P_{jt}^{Q} = \left( \theta(P_{jt}^{V})^{1-\alpha} + (1 - \theta)(P_{jt}^{M})^{1-\alpha} \right)^{1/\gamma} \\
\theta(P_{it}^{V})^{\gamma} Q_{it} = (P_{it}^{V})^{\gamma} V_{it} \\
\theta(P_{jt}^{V})^{\gamma} Q_{jt} = (P_{jt}^{V})^{\gamma} V_{jt} \\
\alpha P_{it}^{V} V_{it} = R_{it} K_{it} \\
\alpha P_{jt}^{V} V_{jt} = R_{jt} K_{jt} \\
(1 - \alpha) P_{it}^{V} V_{it} = W_{it} I_{it} \\
(1 - \alpha) P_{jt}^{V} V_{jt} = W_{jt} I_{jt} \\
(1 - \theta)(P_{it}^{V})^{\gamma} Q_{it} = (P_{jt}^{M})^{\gamma} M_{it} \\
(1 - \theta)(P_{jt}^{V})^{\gamma} Q_{jt} = (P_{jt}^{M})^{\gamma} M_{jt} \\
P_{it}^{M} = (\omega_{it} (P_{it}^{Q})^{1-\phi} + \omega_{d} E_{i,t-1} [P_{it}^{Q}]^{1-\phi} + \omega_{ROW})^{1/\omega_{it}} \\
P_{jt}^{M} = (\omega_{jt} (P_{jt}^{Q})^{1-\phi} + \omega_{d} E_{j,t-1} [P_{jt}^{Q}]^{1-\phi} + \omega_{ROW})^{1/\omega_{jt}} \\
(P_{it}^{M})^{\gamma} M_{it} \omega_{it} = (P_{jt}^{Q})^{\gamma} X_{it} \\
(P_{jt}^{M})^{\gamma} M_{jt} \omega_{jt} = E_{j,t-1} (P_{jt}^{Q})^{\gamma} X_{jt} \\
(P_{jt}^{M})^{\gamma} M_{jt} \omega_{jt} = X_{kit} \\
(P_{jt}^{M})^{\gamma} M_{jt} \omega_{jt} = X_{kit} \\
Q_{it} = C_{it} + I_{it} + X_{it} + X_{ijt} \\
Q_{jt} = C_{jt} + I_{jt} + X_{jt} + X_{ijt} \\
B_{it+1} + \frac{\varphi}{2} (B_{it+1} - B)^2 = (1 + R_{it}^{B}) B_{it} + p_{it}^{Q} X_{ijt} - p_{it}^{Q} X_{ijt} - X_{kit} \\
B_{jt+1} + \frac{\varphi}{2} (B_{jt+1} - B)^2 = (1 + R_{jt}^{B}) B_{jt} + p_{jt}^{Q} X_{ijt} - p_{jt}^{Q} X_{ijt} - X_{kit} \\
K_{it+1} = (1 - \delta) K_{it} + \left( 1 - \frac{\zeta}{2} \frac{I_{it}}{I_{it-1}} - 1 \right)^2 I_{it} \\
K_{jt+1} = (1 - \delta) K_{jt} + \left( 1 - \frac{\zeta}{2} \frac{I_{jt}}{I_{jt-1}} - 1 \right)^2 I_{jt} \\
L_{it} = L_{it} \\
L_{jt} = L_{jt} \\
L_{it}^{\frac{1}{\alpha}} = L_{it}^{\frac{1}{\alpha}} \\
L_{jt}^{\frac{1}{\alpha}} = L_{jt}^{\frac{1}{\alpha}} \\
\Xi_{it} = \beta E_{i} [\Xi_{i,t+1} (1 - \delta) + \Lambda_{it+1} R_{it+1}] \\
\Xi_{jt} = \beta E_{j} [\Xi_{j,t+1} (1 - \delta) + \Lambda_{jt+1} R_{jt+1}] \\
\Lambda_{it} (1 + \varphi (B_{it+1} - B)) = \beta E_{i} \Lambda_{it+1} (1 + R_{it}^{B}) \\
\Lambda_{jt} (1 + \varphi (B_{jt+1} - B)) = \beta E_{j} \Lambda_{jt+1} (1 + R_{jt}^{B}) \\
\Lambda_{it} = \Xi_{it} \left( 1 - \frac{\zeta}{2} \frac{I_{it}}{I_{it-1}} - 1 \right)^2 - \frac{\zeta}{2} \frac{I_{it}}{I_{it-1}} \left( \frac{I_{it}}{I_{it-1}} - 1 \right) = \beta E_{i} \Xi_{i,t+1} \xi \left( \frac{I_{it+1}}{I_{it}} \right)^2 \left( \frac{I_{it+1}}{I_{it}} - 1 \right) \\
\Lambda_{jt} = \Xi_{jt} \left( 1 - \frac{\zeta}{2} \frac{I_{jt}}{I_{jt-1}} - 1 \right)^2 - \frac{\zeta}{2} \frac{I_{jt}}{I_{jt-1}} \left( \frac{I_{jt}}{I_{jt-1}} - 1 \right) = \beta E_{j} \Xi_{j,t+1} \xi \left( \frac{I_{jt+1}}{I_{jt}} \right)^2 \left( \frac{I_{jt+1}}{I_{jt}} - 1 \right)
Symmetric steady state

\[ \omega_s + \omega_d + \omega_{ROW} = 1 \]

\[ V = K^\alpha L^{1-\alpha} \]

\[ \theta Q = V \]

\[ \alpha \theta Q = RK \]

\[ (1 - \alpha) \theta Q = WL \]

\[ (1 - \theta) Q = M \]

\[ M \omega_s = X_{ii} = X_{jj} = X_s \]

\[ M \omega_d = X_{ij} = X_{ji} = X_d \]

\[ M \omega_{ROW} = X_{ki} = X_{kj} = X_{ROW} \]

\[ Q = C + I + (1 - \theta) Q (1 - \omega_{ROW}) \]

\[ BR^B = (1 - \theta) Q \omega_{ROW} \]

\[ \delta K = 1 \]

\[ C^{-\varsigma} = \Lambda \]

\[ L^{-\frac{1}{\alpha}} = \Lambda W \]

\[ 1 = \beta (1 - \delta + R) \]

\[ 1 = \beta (1 + R^B) \]

\[ \Lambda = \Xi \]

\[ \rho^M = 1 \]

\[ \rho^V = 1 \]

\[ \rho^Q = 1 \]

\[ Z = 1 \]

\[ R^B = \frac{1}{\beta} - 1 \]

\[ R = \frac{1}{\beta} - 1 + \delta \]

\[ K \]

\[ \frac{K}{L} = \left( \frac{R}{\alpha} \right)^{\frac{1}{\alpha}} \]

\[ W = (1 - \alpha) \left( \frac{K}{L} \right)^{\alpha} \]

\[ \frac{V}{L} = \left( \frac{K}{L} \right)^{\alpha} = \frac{W}{1 - \alpha} \]

\[ \frac{Q}{L} = \frac{\theta}{\alpha} \]

\[ \frac{C}{L} = \frac{Q}{L} \left( 1 - (1 - \theta) (1 - \omega_{ROW}) \right) - \frac{\delta K}{L} \]

\[ L = \left( \frac{C}{L} \right)^{-\varsigma} W \]

\[ \frac{K}{L} \]

\[ \frac{Q}{L} \]

\[ \frac{V}{L} \]

\[ \frac{C}{L} \]

\[ M = (1 - \theta) Q \]

\[ BR^B = (1 - \theta) Q \omega_{ROW} \]

\[ l = \delta K \]

\[ \Lambda = C^{-\varsigma} \]

\[ M \omega_s = X_{ii} = X_{jj} = X_s \]

\[ M \omega_d = X_{ij} = X_{ji} = X_d \]

\[ M \omega_{ROW} = X_{ki} = X_{kj} = X_{ROW} \]

\[ \Xi = \Lambda \]

In the code,

\[ \rho^M = 1 \]

\[ \rho^V = 1 \]

\[ \rho^Q = 1 \]

\[ Z = 1 \]

\[ R^B = \frac{1}{\beta} - 1 \]

\[ R = \frac{1}{\beta} - 1 + \delta \]

\[ K \]

\[ \frac{K}{L} = \left( \frac{R}{\alpha} \right)^{\frac{1}{\alpha}} \]

\[ W = (1 - \alpha) \left( \frac{K}{L} \right)^{\alpha} \]

\[ \frac{V}{L} = \left( \frac{K}{L} \right)^{\alpha} = \frac{W}{1 - \alpha} \]

\[ \frac{Q}{L} = \frac{\theta}{\alpha} \]

\[ \frac{C}{L} = \frac{Q}{L} \left( 1 - (1 - \theta) (1 - \omega_{ROW}) \right) - \frac{\delta K}{L} \]

\[ L = \left( \frac{C}{L} \right)^{-\varsigma} W \]

\[ \frac{K}{L} \]

\[ \frac{Q}{L} \]

\[ \frac{V}{L} \]

\[ \frac{C}{L} \]

\[ M = (1 - \theta) Q \]

\[ BR^B = (1 - \theta) Q \omega_{ROW} \]

\[ l = \delta K \]

\[ \Lambda = C^{-\varsigma} \]

\[ M \omega_s = X_{ii} = X_{jj} = X_s \]

\[ M \omega_d = X_{ij} = X_{ji} = X_d \]

\[ M \omega_{ROW} = X_{ki} = X_{kj} = X_{ROW} \]

\[ \Xi = \Lambda \]
Log-linearization

\[ v_{it} = z_{it} + \alpha k_{it} + (1 - \alpha)\delta_{it} \]
\[ v_{jt} = z_{jt} + \alpha k_{jt} + (1 - \alpha)\delta_{jt} \]
\[ q_{it} = \theta v_{it} + (1 - \theta)\mu_{it} \]
\[ q_{jt} = \theta v_{jt} + (1 - \theta)\mu_{jt} \]
\[ n_{jt}^Q + q_{jt} = n_{jt}^V + v_{jt} \]
\[ n_{jt}^Q + q_{jt} = n_{jt}^V + v_{jt} \]
\[ p_{jt}^V + v_{jt} = r_{jt} + k_{jt} \]
\[ p_{jt}^V + v_{jt} = r_{jt} + k_{jt} \]
\[ p_{jt}^V + v_{jt} = r_{jt} + k_{jt} \]
\[ p_{jt}^V + v_{jt} = r_{jt} + k_{jt} \]
\[ p_{jt}^Q + q_{jt} = m_{jt} + \eta_{jt}^M \]
\[ p_{jt}^Q + q_{jt} = m_{jt} + \eta_{jt}^M \]
\[ p_{jt}^Q = \omega_{jt} p_{jt}^Q + \omega_{jt} E_{jt-1}[p_{jt}^Q] \]
\[ p_{jt}^Q = \omega_{jt} E_{jt-1}[p_{jt}^Q] + \omega_{jt} p_{jt}^Q \]
\[ \phi_{jt}^M + m_{jt} = \phi_{jt}^Q + \bar{x}_{jt} \]
\[ \phi_{jt}^M + m_{jt} = \phi_{jt}^Q + \bar{x}_{jt} \]
\[ \phi_{jt}^M + m_{jt} = \phi_{jt}^Q + \bar{x}_{jt} \]
\[ \phi_{jt}^M + m_{jt} = \phi_{jt}^Q + \bar{x}_{jt} \]
\[ \phi_{jt}^M + m_{jt} = \phi_{jt}^Q + \bar{x}_{jt} \]
\[ \phi_{jt}^M + m_{jt} = \phi_{jt}^Q + \bar{x}_{jt} \]
\[ \phi_{jt}^M + m_{jt} = \phi_{jt}^Q + \bar{x}_{jt} \]
\[ Q_{jt} = C_{jt} + \bar{x}_{jt} X_{jt} \bar{x}_{jt} + X_{jt} \bar{x}_{jt} \]
\[ Q_{jt} = C_{jt} + \bar{x}_{jt} X_{jt} \bar{x}_{jt} + X_{jt} \bar{x}_{jt} \]
\[ b_{jt} + (1 + R^i) h_{jt} + X_{jt} \bar{x}_{jt} - p_{jt}^Q - x_{jt} - X_{jt} \bar{x}_{jt} = X_{jt} \bar{x}_{jt} \]
\[ b_{jt} + (1 + R^i) h_{jt} + X_{jt} \bar{x}_{jt} - p_{jt}^Q - x_{jt} - X_{jt} \bar{x}_{jt} = X_{jt} \bar{x}_{jt} \]
\[ k_{jt} + (1 - \delta) h_{jt} + i_{jt} \delta \]
\[ k_{jt} + (1 - \delta) h_{jt} + i_{jt} \delta \]
\[ -c_{jt} = \lambda_{jt} + p_{jt}^Q \]
\[ -c_{jt} = \lambda_{jt} + p_{jt}^Q \]
\[ 1 \bar{\psi} = \lambda_{jt} + w_{jt} \]
\[ 1 \bar{\psi} = \lambda_{jt} + w_{jt} \]
\[ \lambda_{jt} + \phi_{jt} + E_{jt} = \lambda_{jt+1} \]
\[ \lambda_{jt} + \phi_{jt} + E_{jt} = \lambda_{jt+1} \]
\[ p_{jt}^Q + \lambda_{jt} - \zeta_{jt} + \zeta_{jt} = \beta E_{jt}[s_{jt+1} - s_{jt}] \]
\[ p_{jt}^Q + \lambda_{jt} - \zeta_{jt} + \zeta_{jt} = \beta E_{jt}[s_{jt+1} - s_{jt}] \]
\[ \begin{bmatrix} z_{jt} \\ s_{jt} \end{bmatrix} = \begin{bmatrix} \rho_s & \rho_d \\ \rho_d & \rho_s \end{bmatrix} \begin{bmatrix} z_{jt-1} \\ s_{jt-1} \end{bmatrix} + \begin{bmatrix} \sigma_s & \sigma_d \\ \sigma_d & \sigma_s \end{bmatrix} \begin{bmatrix} v_{jt} \\ \phi_{jt} \end{bmatrix} \]

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